Math 4355 – Spring 2021 Name:

TEST #1 - version 2

Please, show your work, justify every step and write legibly. When you are done, scan, save the file as LASTNAME_FIRSTNAME_T1.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. NOTE: You need to send your email before 11:30AM on Feb 25 to receive credit.

(1)[4 Pts]

(a) Let $V = \mathbb{R}^3$ and consider the subspace $V_0 \subset V$ given by

$$V_0 = \operatorname{span}\{(-1, 1, -1)\}.$$

Find the orthogonal complement $V_0^\perp$ of V_0 in V and find and an orthonormal basis of the space $V_0^\perp$.

(b) Let $V = \mathbb{R}^4$ and consider the subspace $V_1 \subset V$ given by

$$V_1 = \operatorname{span}\{(-1, 1, -1, 0)\}.$$

Find the orthogonal complement $V_1^\perp$ of V_1 in V and find and an orthonormal basis of the space $V_1^\perp$. [Hint: Use part (a)]

(2)[4 Pts] Consider the inner product space $V = L^2([-\pi, \pi])$ with the standard inner product. Compute the orthogonal projection of the function

$$f(x) = \begin{cases} 1 & \text{if } -\pi \le x < 0 \\ -1 & \text{if } \quad 0 \le x \le \pi \end{cases}$$

onto the subspace $V_0 = \operatorname{span}\{1, \sin x, \cos x, \sin 2x, \cos 2x\} \subset L^2([-\pi, \pi])$].

[Hint: It may be useful to use even and odd properties.]

(3)[5 Pts] Consider the sequence of functions $(f_n) \subset L^2([0, 1])$ defined by

$$f_n(x) = \begin{cases} 1 - (nx)^2 & 0 \le x < \frac{1}{n} \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $f_n(x)$ for $n = 1, 2, 3$.

(b) Prove that the sequence (f_n) converges to the function $f(x) = 0$, $x \in [0, 1]$, in the L^2 norm.

(c) Show that the sequence $(f_n(x))$ does not converge pointwise to the functions $f(x) = 0$, $x \in [0, 1]$ [Hint: consider the point $x = 0$]