Math 4355 – Spring 2021 Name:

TEST #2 - version 1

Please, show your work, justify every step and write legibly. When you are done, scan, save the file as LASTNAME_FIRSTNAME_T2.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. NOTE: You need to send your email before 11:30AM on March 25 to receive credit.

- (1) [8 Pts]

(a) Compute the sine series of $f(x) = \sin x$, on the interval $0 \leq x \leq \pi$.

(b) Does the sine series of f converge uniformly to f in $[0, \pi]$? If not, where does it fail to converge? Justify.

(c) Without computing the Fourier coefficients, set up the computation of the cosine series of $f(x) = \sin x$, on the interval $0 \leq x \leq \pi$.

(d) Does the cosine series of f converge uniformly to f in $[0, \pi]$? If not, where does it fail to converge? Justify.

- (2) [8 Pts] Consider the function

$$f(x) = \begin{cases} -1 & \text{if } -\pi \leq x \leq -\frac{\pi}{4} \\ 0 & \text{if } -\frac{\pi}{4} < x < \frac{\pi}{4} \\ 1 & \text{if } \frac{\pi}{4} \leq x \leq \pi \end{cases}$$

(a) Sketch the function.

(b) Compute the Fourier series of f, valid in the interval $[-\pi, \pi]$. (Hint: Take advantage of even/odd properties to reduce calculations.)

(c) Does the Fourier series converge to f uniformly on $[-\pi \leq x \leq \pi]$? If not, where does it fail to converge? Justify.

- (3) [4 Pts]

(a) Compute the Fourier series of $f(x) = \sin^2 x$ valid in the interval $[-\pi, \pi]$

(b) Compute the Complex Fourier series of $f(x) = \sin^2 x$ valid in the interval $[-\pi, \pi]$ [Hint: there is no need of integration. You can derive it using the result of part (a).]