Math 4355 – Spring 2021 Name:

TEST #3 - version 1

Please, show your work, justify every step and write legibly. When you are done, scan, save the file as LASTNAME_FIRSTNAME_T3.pdf and email to `dlabate@math.uh.edu` or `dlabate@uh.edu`. NOTE: You need to send your email before 11:30AM on April 20 to receive credit.

- (1) [3 Pts] Let $A > 0$ be a fixed number. Does the following functions define a causal filter? Justify your answer.

(a) $h_1(t) = \begin{cases} e^{At} & \text{if } t < 0; \\ e^{-At} & \text{if } t \geq 0. \end{cases}$

(b) $h_2(t) = \begin{cases} 0 & \text{if } t < 0; \\ e^{-At} & \text{if } t \geq 0. \end{cases}$

- (2) [9 Pts] Let $f(t) = \begin{cases} 1 & \text{if } -\pi \leq t \leq \pi; \\ 0 & \text{otherwise.} \end{cases}$

We found that its Fourier transform is $\hat{f}(\omega) = \sqrt{\frac{2}{\pi}} \frac{\sin(\pi\omega)}{\omega}$.

(a) Use the Fourier transform of f and the properties of the Fourier transform to compute the Fourier transform $\hat{c}(\omega)$ of the function $c(t) = (f * f)(t)$

(b) Use the Fourier transform of f and the properties of the Fourier transform to compute the Fourier transform of

$$h(t) = \begin{cases} 1 & \text{if } -1 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(c) Use the Fourier transform of h and the properties of the Fourier transform to compute the Fourier transform of

$$g(t) = \begin{cases} t^2 & \text{if } -1 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

- (3) [6 Pts] Let

$$f(t) = \begin{cases} 1 & 0 \leq t \leq 2 \\ 0 & \text{otherwise;} \end{cases} \qquad g(t) = \begin{cases} t & 0 \leq t \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $h(t) = (f * g)(t)$.
(b) Sketch the graph of f, g, h over the interval $[-1, 4]$.