Math 6320 – Fall 2012 Name:

HW #1

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[9 Pts] Let E be a subset of a metric space $\mathbb{R}^n$.

(i) E is *sequentially compact* if every sequence $\{x_n\}$ of points from E contains a convergent subsequence $\{x_{n_k}\}$ whose limit belongs to E.

(ii) E is *complete* if every Cauchy sequence in E converges to a point in E.

(iii) E is *totally bounded* if given any $r > 0$ we can cover E by finitely many open balls of radius r. That is, for each $r > 0$ there must exist finitely many points $x_1, \ldots, x_N \in \mathbb{R}^n$ such that

$$E \subset \bigcup_{k=1}^{N} B(x_k, r).$$

Prove that the following are equivalent.

(a) E is compact.

(b) E is sequentially compact.

(c) E is complete and totally bounded.

[Hint. Prove $(a) \to (b)$, $(b) \to (c)$, $(c) \to (b)$, to prove $(b) \to (a)$ use $(b) \to (c)$.]

(2)[3 Pts] Solve Problem 35 (p.14) in Textbook.