

HW #2

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[4 Pts] Solve Problem 9, p.35, in textbook.

(2)[5 Pts] Let E be a subset of \mathbb{R}^n .

(i) Prove that if E has non-empty interior, then $\lambda^*(E) > 0$.

(ii) Prove that, if $\lambda^*(E) = 0$, then $\mathbb{R}^n \setminus E$ is dense in \mathbb{R}^n . Recall that a set A is *dense* in \mathbb{R}^n if $\overline{A} = \mathbb{R}^n$.

(3)[9 Pts] Let $C \subset [0, 1]$ be the Cantor ternary set. Prove that (a) C is closed; (b) C contains no open intervals; (c) C is uncountable; (d) every point of C is a limit point of C ; (e) for any point $z \in [0, 2]$, there exist $x, y \in C$ such that $z = x + y$ (this is Problem 21, p.42).