Math 6320 – Fall 2012 Name:

HW #5

Please, write clearly and justify all your steps, to get proper credit for your work.

(1)[4 Pts] Let $f : \mathbb{R}^d \mapsto [-\infty, \infty]$. Prove that, if there exists a continuous function $g : \mathbb{R}^d \mapsto \mathbb{R}$ which is equal to f except for possibly a null set, then f is Lebesgue measurable.

(2)[4 Pts] Let $E \subset \mathbb{R}^d$. Show that $f : E \mapsto \mathbb{R}$ is $\mathcal{M}$-measurable if and only if $\{x \in E : f(x) \leq r\}$ is $\mathcal{M}$-measurable for each rational $r \in \mathbb{Q}$.

(3) [4 Pts] Let $E \subset \mathbb{R}^d$ and suppose that $f : E \mapsto \mathbb{R}$ is such that $|f|$ is Lebesgue measurable. Does it follow that f is Lebesgue measurable?

(4)[4 Pts] Problem 23 from Ch. 5

(5)[4 Pts] Problem 1 from Ch. 6.