Math 6320 – Fall 2012 Name:

HW #6

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) [6 Pts] Let $E \subset \mathbb{R}^n$ be a Lebesgue measurable set, and let f_n, f, g_n, g be measurable functions on E that are finite a.e. Prove the following statements.

(a) If $f_n \xrightarrow{m} f$ and $f_n \xrightarrow{m} g$, then $f = g$ a.e.

(b) If $f_n \xrightarrow{m} f$ and $g_n \xrightarrow{m} g$, then $f_n + g_n \xrightarrow{m} f + g$.

(c) If $\lambda(E) < \infty$, $f_n \xrightarrow{m} f$ and $f_n \xrightarrow{m} g$, then $f_n\, g_n \xrightarrow{m} f\, g$.

(d) Show that the conclusion of part (c) may fail if $\lambda(E) = \infty$.

(e) If $\lambda(E) < \infty$, and $f_n \to f$ a.e. on E, then $f_n \xrightarrow{m} f$.

(f) Show that the conclusion of part (e) may fail if $\lambda(E) = \infty$.

(2) [4 Pts] Let $E \subset \mathbb{R}^n$ be a Lebesgue measurable set and f, f_n be measurable functions on E that are finite a.e. such that $f_n \xrightarrow{m} f$. Prove that there exist a subsequence of f_n converging to f a.e.

(3) [4 Pts] Show that the conclusion of the LDCT continues to hold if we replace the hypothesis $f_n \to f$ a.e. with $f_n \xrightarrow{m} f$. [Hint: use the conclusion of problem (2)]

(4) [4Pts] Solve Problem 9 from Chapter 7.