Math 6320 – Fall 2012 Name:

HW #7

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) [4 Pts] Let $f : E \to [0, \infty]$ be a Lebesgue measurable function, where $E \subset \mathbb{R}^n$ is a Lebesgue measurable set. Let $1 < p < \infty$. Prove that, for each number $\alpha > 0$,

$$\lambda\left(\{x : |f(x)| > \alpha\}\right) \leq \frac{1}{\alpha^p} \int_{|f|>\alpha} |f|^p \, d\lambda.$$

(2) [4 Pts] Show that if $f \in L^1(\mathbb{R})$ is differentiable at $x = 0$ and $f(0) = 0$, then

$$\int_{-\infty}^{\infty} \frac{f(x)}{x} \, dx$$

exists

(4) [4Pts] Solve Problem 21 from Chapter 7.

(4) [4Pts] Solve Problem 12 from Chapter 8.