

HW #1

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) [5 Pts]

- (a) Let $f \in C_c(\mathbb{R})$ (i.e., f is continuous with compact support). Prove that $f \in L^p(\mathbb{R})$ for every $1 \leq p < \infty$.
- (b) Show that $C_0(\mathbb{R})$ (i.e., the set of continuous functions vanishing at ∞) is not contained in $L^p(\mathbb{R})$ for any $1 \leq p < \infty$.
- (c) Although $C_0(\mathbb{R})$ is not contained in $L^p(\mathbb{R})$, $1 \leq p < \infty$, nor conversely, the intersection of $C_0(\mathbb{R})$ and $L^p(\mathbb{R})$ is larger than $C_c(\mathbb{R})$. Find an example of a function $f \in C_0(\mathbb{R}) \cap L^p(\mathbb{R})$ which has no compact support.

(2) [4 Pts] Solve Problem 28 and 29 from Chapter 6.

(3) [2 Pts] Solve Problem 43 from Chapter 6.