Math 6321 – Spring 2013 Name:

HW #3

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) [3 Pts] Let $f_a = f * \phi_a$, where $f \in L^\infty(\mathbb{R}^n)$, $\phi \in L^1(\mathbb{R}^n)$, $\int_{\mathbb{R}^n} \phi = 1$ and $\phi_a(x) = a^{-n}\phi(\frac{x}{a})$. Prove that $\lim_{a\to 0} |f_a(x) - f(x)| = 0$ at every point of continuity of f.

(2) [3 Pts] Let $\phi(x) = \frac{1}{\pi}\left(\frac{\sin x}{x}\right)^2$, where $x \in \mathbb{R}$. Let $\phi_a(x) = a^{-n}\phi(\frac{x}{a})$. Prove that, if $f \in L^1(\mathbb{R})$ and if f is continuous at x, then

$$\lim_{a\to 0} f * \phi_a(x) = f(x).$$

(3) [4 Pts] Let H be a Hilbert space and $M \subset H$ be a subspace. The *orthogonal complement* of M is the set

$$M^\perp\{x \in H : \langle x, y\rangle = 0,\, y \in M\}.$$

(1) Prove that $M^\perp$ is a closed subspace of H.

(2) Let M be the set of all even functions in $L^2(\mathbb{R})$. Show that M is a closed subspace of $L^2(\mathbb{R})$, and its orthogonal complement is the set of odd functions in $L^2(\mathbb{R})$.

(4) [3 Pts] Solve Problem 12 from Chapter 12.

(5) [3 Pts] Solve Problem 16 from Chapter 12.