Math 6321 – Spring 2013 Name:

HW #4

Please, write clearly and justify all your steps, to get proper credit for your work.

(1) [3 Pts] Let $x^k f(x)$ be in $L^1(\mathbb{R})$, for $k = 0, 1, \ldots, n$. Prove that $\hat{f}$ is n times differentiable and

$$[(-ix)^k f]^\wedge(\xi) = \frac{d^k \hat{f}}{d\xi^k}(\xi), \qquad \text{for } k = 1, 2, \ldots, n.$$

(2) [3 Pts] Suppose that $f \in C^n(\mathbb{R}) \cap L^1(\mathbb{R})$ and that all derivatives $\frac{d^k f}{dx^k}(x)$, $k = 1, \ldots, n$ are in $L^1(\mathbb{R})$. Prove that

$$[\frac{d^k f}{dx^k}]^\wedge(\xi) = (i\xi)^k \hat{f}(\xi), \qquad \text{for } k = 1, 2, \ldots, n.$$

(3) [2 Pts] Prove that if $f \in L^1(\mathbb{R})$ and has bounded support then $\hat{f} \in C^\infty(\mathbb{R})$.

(4) [3 Pts] Prove that if $f \in C^2(\mathbb{R})$ and if f, f', f'' are in $L^1(\mathbb{R})$, then $\hat{f} \in L^1(\mathbb{R})$.