

Homework #1

Please, write clearly and justify your work to receive credit.

(1) [6 Pts] *The following problem is about calculating the number of parameters of a feedforward neural network.*

(a) *Derive a formula to compute the number of parameters between 2 (fully connected) layers of a feedforward neural network where the first layer has N_ℓ neurons and the second layer has $N_{\ell+1}$ neurons.*

(b) *Determine the number of parameters of a feedforward neural network with the following architecture:*

- *Input layer: 3 neurons.*
- *Hidden layer 1: 12 neurons.*
- *Hidden layer 2: 8 neurons.*
- *Hidden layer 3: 4 neurons.*
- *Output layer: 2 neurons.*

(c) *Design a feedforward neural network with input dimension 2, output dimension 1 and at least 2 hidden layers which contains at least 400 but no more than 405 parameters.*

(2) [6 Pts] *This problem is about using ReLU feedforward neural networks to implement piecewise linear functions.*

(a) *Design a shallow neural network with ReLU activation function implementing the following function*

$$T(x) = \begin{cases} 3x & \text{if } 0 \leq x < \frac{1}{3} \\ \frac{3}{2}(1-x) & \text{if } \frac{1}{3} \leq x \leq 1 \end{cases} \quad x \in \mathbb{R},$$

(b) *Design a shallow neural network with ReLU activation function implementing a piecewise linear function on \mathbb{R} with 2 nodes, that is, a piecewise linear function that changes slope exactly 3 times.*

(c) *Derive a formula relating the number of parameters M of a shallow neural network with ReLU activation function implementing a piecewise linear function on \mathbb{R} to the number of nodes K of the function.*