[*] means the paper was already selected by a student

[*] “A two-step Hilbert transform method for 2D image reconstruction” by Frédéric Noo et al, in 2004 Phys. Med. Biol. 49 3903, doi:10.1088/0031-9155/49/17/006

Abstract: The paper describes a new accurate two-dimensional (2D) image reconstruction method consisting of two steps. In the first step, the backprojected image is formed after taking the derivative of the parallel projection data. In the second step, a Hilbert filtering is applied along certain lines in the differentiated backprojection (DBP) image. Formulae for performing the DBP step in fan-beam geometry are also presented. The advantage of this two-step Hilbert transform approach is that in certain situations, regions of interest (ROIs) can be reconstructed from truncated projection data. Simulation results are presented that illustrate very similar reconstructed image quality using the new method compared to standard filtered backprojection, and that show the capability to correctly handle truncated projections. In particular, a simulation is presented of a wide patient whose projections are truncated laterally yet for which highly accurate ROI reconstruction is obtained.

http://iopscience.iop.org/0031-9155/49/17/006

[*] “The meaning of interior tomography” by Ge Wang and Hengyong Yu, in 2013 Phys. Med. Biol. 58 R161 doi:10.1088/0031-9155/58/16/R161

Summary: While traditional tomographic methods require that complete data be acquired from a whole cross-section of interest, interior tomography is an emerging area for theoretically exact reconstruction of a small region of interest (ROI) inside a large object from truncated data that directly involves the ROI. Interior tomography was proven valid, aided by a known sub-region in 2007 and a piecewise well-behavior model in 2009. Then, interior tomography was extended to SPECT, MRI, phase-contrast tomography, and so on. Consequently, interior tomography has been elevated to a general tomographic principle. Now, we envision that the next stage of biomedical imaging will be the grand fusion of many tomographic modalities into a single gantry (‘all in one’) for simultaneous imaging of many complementary features (‘all at once’). This integration is referred to as ‘omni-tomography’, potentially instrumental for development of systems biology and personalized medicine, since many physiological processes are complicated, dynamic, and correlated.

http://iopscience.iop.org/0031-9155/58/16/R161/pdf/0031-9155_58_16_R161.pdf

[*] “Inversion of the attenuated Radon transform” by Natterer in 2001 Inverse Problems 17 113 doi:10.1088/0266-5611/17/1/309

Abstract. We derive an exact inversion formula for the attenuated Radon transform. The formula is closely related to Novikov's inversion formula, but our derivation is completely different. We also give an implementation of the inversion formula very similar to the filtered backprojection algorithm of x-ray tomography and present numerical results.

http://iopscience.iop.org/0266-5611/17/1/309

“On local tomography”, by P Kuchment et al,1995 Inverse Problems 11 571 doi:10.1088/0266-5611/11/3/006

Abstract. In this paper we explain how the local tomography approach to tomographic problems can be extended to a wide range of situations including limited data problems, attenuated transforms, and generalized radon transforms. Numerical examples illustrate the use of local tomography applied to complete and limited data problems. Our analytic results are obtained through the use of microlocal analysis.

http://iopscience.iop.org/0266-5611/11/3/006

[*] “Local Tomography”, by Adel Faridani, Erik L. Ritman, and Kennan T. Smith, SIAM J. Appl. Math., 52(2), 459–484. DOI:10.1137/0152026

Abstract. Tomography produces the reconstruction of a function f from a large number of line integrals of f. Conventional tomography is a global procedure in that the standard convolution formulas for reconstruction at a single point require the integrals over all lines within some plane containing the point. Local tomography, as introduced initially, produced the reconstruction of the related function Λf, where $\Lambda $ is the square root of $ - \Delta $, the positive Laplace operator. The reconstruction of Λf is local in that reconstruction at a point requires integrals only over lines passing infinitesimally close to the point, and Λf has the same smooth regions and boundaries as f. However, Λf is cupped in regions where f is constant. $\Lambda ^{ - 1} f$, also amenable to local reconstruction, is smooth everywhere and contains a counter-cup. This article provides a detailed study of the actions of $\Lambda $ and $\Lambda ^{ - 1} $, and shows several examples of what can be achieved with a linear combination. It includes the results of x-ray experiments in which the line integrals are obtained from attenuation measurements on two-dimensional image intensifiers and fluorescent screens, instead of the usual linear detector arrays.

http://epubs.siam.org/doi/abs/10.1137/0152026

[*] "A large class of inversion formulae for the 2D Radon transform of functions of compact support." By R.Clackdoyle, F.Noo. in 2004 Inverse Problems 20, 1281-1291

Abstract. We demonstrate that there are many direct inversion formulae for the two-dimensional (2D) Radon transform that are not equivalent to Radon's original inversion formula. Our explicit formulae have a form similar to classical filtered backprojection but do require some information about the support of the object function. We present some numerical results to illustrate significant differences in reconstructions as alternate formulae are applied to noisy data.

http://iopscience.iop.org/0266-5611/20/4/016

"Truncated Hilbert transform and image reconstruction from limited tomographic data." By M.Defrise, F.Noo, R.Clackdoyle, H.Kudo. 2006 Inverse Problems 22, 1037-1053

Abstract. A data sufficiency condition for 2D or 3D region-of-interest (ROI) reconstruction from a limited family of line integrals has recently been introduced using the relation between the backprojection of a derivative of the data and the Hilbert transform of the image along certain segments of lines covering the ROI. This paper generalizes this sufficiency condition by showing that unique and stable reconstruction can be achieved from an even more restricted family of data sets, or, conversely, that even larger ROIs can be reconstructed from a given data set. The condition is derived by analysing the inversion of the truncated Hilbert transform, here defined as the problem of recovering a function of one real variable from the knowledge of its Hilbert transform along a segment which only partially covers the support of the function but has at least one end point outside that support. A proof of uniqueness and a stability estimate are given for this problem. Numerical simulations of a 2D thorax phantom are presented to illustrate the new data sufficiency condition and the good stability of the ROI reconstruction in the presence of noise.

http://iopscience.iop.org/0266-5611/22/3/019

“Tomography, Approximate Reconstruction, and Continuous Wavelet Transforms”, by W.R. Madych, in Applied and Computational Harmonic Analysis, Volume 7, Issue 1, July 1999, Pages 54–100.

Abstract. It has been recognized for some time now that certain high-frequency information concerning planar densities f in a neighborhood of a point can be recovered from data which consist of averages of f over lines that are relatively close to that point. The wavelet transform of f is a classical tool for analyzing local frequency content. In this article we introduce continuous wavelet transforms which are particularly well suited to producing high-resolution local reconstructions from local data of the type described above. We also show how such transforms can be realized numerically via simple modifications of well-established convolution backprojection-type algorithms.As part of our development we review the concepts of “local tomography” and “pseudolocal tomography” introduced by several authors and indicate that, in effect, these notions basically involve the computation of a wavelet transform. The results in this paper are based on the observation that Radon's classical inversion formula is a summability formula with an integrable convolution-type summability kernel.

http://www.sciencedirect.com/science/article/pii/S1063520398902585

“Reconstructing singularities of a function from its Radon transform” by A.G. Ramm and A.I. Zaslavskya, in Mathematical and Computer Modelling,Volume 18, Issue 1, July 1993, Pages 109–138

Abstract. We study the relation between the singularities of a function f and its Radon transform R(f). We prove that their singular loci are related via Legendre transform. Geometric properties of the singular locus of R(f) are studied. The problem of computing the Legendre transform from approximately known data is discussed.

http://www.sciencedirect.com/science/article/pii/089571779390083B

“The interior Radon transform”, by P. Maass, in SIAM Appl. Math. 52 (1992), pp. 710-724

Abstract. The interior Radon transform arises from a limited data problem in computerized tomography when only rays travelling through a specified region of interest are measured. This problem occurs due to technical restrictions of the sampling apparatus or in an endeavour to reduce the X-ray dose. The corresponding operator $\mathcal{R}_I $ is investigated as a mapping between weighted $L_2 $-spaces. The main result is a singular value decomposition (SVD) for this operator for functions of unbounded support in $\mathbb{R}^2 $. The proof is based on the construction of intertwining differential operators. The techniques used are unified in the sense that SVDs for other Radon transforms with rotational symmetry can easily be derived in the same way. Consistency conditions are also obtained for $\mathcal{R}_I $ for functions of bounded and unbounded support. Many of the results generalize to higher dimensions; they are stated whenever they follow directly from the two-dimensional case.

http://epubs.siam.org/doi/abs/10.1137/0152040

“Finite Hilbert transform with incomplete data: null-space and singular values”, by Katsevich and A Tovbis 2012 Inverse Problems 28 105006 doi:10.1088/0266-5611/28/10/105006

Abstract. Using the Gelfand–Graev formula, the interior problem of tomography reduces to the inversion of the finite Hilbert transform (FHT) from incomplete data. In this paper, we study several aspects of inverting the FHT when the data are incomplete. Using the Cauchy transform and an approach based on the Riemann–Hilbert problem, we derive a differential operator that commutes with the FHT. Our second result is the characterization of the null-space of the FHT in the case of incomplete data. Also, we derive the asymptotics of the singular values of the FHT in three different cases of incomplete data.

http://iopscience.iop.org/0266-5611/28/10/105006?rel=ref&relno=6

[*] “An Inversion Formula for Cone-Beam Reconstruction”, by Heang K. Tuy, SIAM J. Appl. Math., 43(3), 546–552.

Abstract. An analytic inversion formula allowing the reconstruction of a three-dimensional object from x-ray cone-beams is given. The formula is valid for the case where the source of the beams describes a bounded curve satisfying a set of weak conditions.

http://epubs.siam.org/doi/abs/10.1137/0143035

“Singularities of the X-Ray Transform and Limited Data Tomography in $\mathbb{R}^2 $ and $\mathbb{R}^3 $”, by Eric Todd Quinto, SIAM Journal on Mathematical Analysis, 1993, Vol. 24, No. 5, pp. 1215-1225

Abstract. Given a function f, the author specifies the singularities of f that are visible in a stable way from limited X-ray tomographic data. This determines which singularities of f can be stably recovered from limited data and which cannot, no matter how good the inversion algorithm. Microlocal analysis is used to determine the relationship between the singularities of a function f and those of its X-ray transform. The results are applied to determine the singularities that are visible for limited angle tomography and the interior and exterior problems. The author also suggests a practical method to use this relationship to reconstruct singularities of f from limited data Rf. The X-ray transform with sources on a curve in $\mathbb{R}^3 $ is also analyzed.

http://epubs.siam.org/doi/abs/10.1137/0524069

“Wavelet methods for a weighted sparsity penalty for region of interest tomography”, by Esther Klann et al 2015 Inverse Problems 31 025001 doi:10.1088/0266-5611/31/2/025001

Abstract. We consider region of interest (ROI) tomography of piecewise constant functions. Additionally, an algorithm is developed for ROI tomography of piecewise constant functions using a Haar wavelet basis. A weighted ℓp–penalty is used with weights that depend on the relative location of wavelets to the region of interest. We prove that the proposed method is a regularization method, i.e., that the regularized solutions converge to the exact piecewise constant solution if the noise tends to zero. Tests on phantoms demonstrate the effectiveness of the method.

http://iopscience.iop.org/0266-5611/31/2/025001/

http://equinto.math.tufts.edu/research/KlannQuintoRamlau_waveletROI.pdf

“The attenuated x-ray transform: recent developments”, by D. V. Finch, pp. 47-66 in Inside Out, Camb. U. Press, 2003.

Abstract. We survey recent work on the attenuated x-ray transform, concentrating especially on the inversion formulas found in the last few years

http://math.oregonstate.edu/~finch/papers/attenpost.pdf

“Nonuniqueness in Inverse Radon Problems: The Frequency Distribution of the Ghosts.” By Alfred K. Louis, in Mathematische Zeitschrift (1984), Volume: 185, page 429-440

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http://www.digizeitschriften.de/download/PPN266833020_0185/log37.pdf