Math 7320 – Fall 2010 Name:

HW #1

Please, write clearly and justify your arguments to get credit for your work.

(1)[1 Pts] Show that the Triangle Inequality implies

$$|d(x,y) - d(z,w)| \leq d(x,z) + d(y,w)$$

(2)[2 Pts] Find a sequence which converges to 0 but is not in any space ℓ^p, where $1 \leq p < \infty$.

(3)[3 Pts] Let $p \geq 1$. Is it true that $\ell^p \subset \ell^{p+1}$? Or that $\ell^{p+1} \subset \ell^p$? Justify your answers.

(4)[4 Pts] Let (X,d) be a metric space. The *diameter* $\delta(A)$ of a nonempty set $A \subset X$ is defined by

$$\delta(A) = \sup_{x,y \in A} d(x,y).$$

A is said to be bounded if $\delta(A) < \infty$.

(a) Show that $A \subset B$ implies that $\delta(A) \leq \delta(B)$.

(b) Show that the union of two bounded subsets of X is a bounded set.