The exact probabilities are .2622 and .8348, respectively, so the approximations are quite good. In the last calculation, the probability $P(5 \le X \le 15)$ is being approximated by the area under the normal curve between 4.5 and 15.5—the continuity correction is used for both the upper and lower limits. ■

When the objective of our investigation is to make an inference about a population proportion p, interest will focus on the sample proportion of successes X/n rather than on X itself. Because this proportion is just X multiplied by the constant $1/n$, it will also have approximately a normal distribution (with mean $\mu = p$ and standard deviation $\sigma = \sqrt{pq/n}$) provided that both $np \ge 10$ and $nq \ge 10$. This normal approximation is the basis for several inferential procedures to be discussed in later chapters.

EXERCISES Section 4.3 (28–58)

28. Let Z be a standard normal random variable and calculate the following probabilities, drawing pictures wherever appropriate.
a. $P(0 \le Z \le 2.17)$ **b.** $P(0 \le Z \le 1)$
c. $P(-2.50 \le Z \le 0)$ **d.** $P(-2.50 \le Z \le 2.50)$
e. $P(Z \le 1.37)$ **f.** $P(-1.75 \le Z)$
g. $P(-1.50 \le Z \le 2.00)$ **h.** $P(1.37 \le Z \le 2.50)$
i. $P(1.50 \le Z)$ **j.** $P(|Z| \le 2.50)$

29. In each case, determine the value of the constant c that makes the probability statement correct.
a. $\Phi(c) = .9838$ **b.** $P(0 \le Z \le c) = .291$
c. $P(c \le Z) = .121$ **d.** $P(-c \le Z \le c) = .668$
e. $P(c \le |Z|) = .016$

30. Find the following percentiles for the standard normal distribution. Interpolate where appropriate.
a. 91st **b.** 9th **c.** 75th
d. 25th **e.** 6th

31. Determine z_α for the following:
a. $\alpha = .0055$ **b.** $\alpha = .09$
c. $\alpha = .663$

32. Suppose the force acting on a column that helps to support a building is a normally distributed random variable X with mean value 15.0 kips and standard deviation 1.25 kips. Compute the following probabilities by standardizing and then using Table A.3.
a. $P(X \le 15)$ **b.** $P(X \le 17.5)$
c. $P(X \ge 10)$ **d.** $P(14 \le X \le 18)$
e. $P(|X - 15| \le 3)$

33. Mopeds (small motorcycles with an engine capacity below 50 cm^3) are very popular in Europe because of their mobility, ease of operation, and low cost. The article "Procedure to Verify the Maximum Speed of Automatic Transmission Mopeds in Periodic Motor Vehicle Inspections" (*J. of Automobile Engr.*, 2008: 1615–1623) described a rolling bench test for determining maximum vehicle speed. A normal distribution with mean value 46.8 km/h and standard deviation 1.75 km/h is postulated. Consider randomly selecting a single such moped.
a. What is the probability that maximum speed is at most 50 km/h?
b. What is the probability that maximum speed is at least 48 km/h?
c. What is the probability that maximum speed differs from the mean value by at most 1.5 standard deviations?

34. The article "Reliability of Domestic-Waste Biofilm Reactors" (*J. of Envir. Engr.*, 1995: 785–790) suggests that substrate concentration (mg/cm^3) of influent to a reactor is normally distributed with $\mu = .30$ and $\sigma = .06$.
a. What is the probability that the concentration exceeds .25?
b. What is the probability that the concentration is at most .10?
c. How would you characterize the largest 5% of all concentration values?

35. Suppose the diameter at breast height (in.) of trees of a certain type is normally distributed with $\mu = 8.8$ and $\sigma = 2.8$, as suggested in the article "Simulating a Harvester-Forwarder Softwood Thinning" (*Forest Products J.*, May 1997: 36–41).
a. What is the probability that the diameter of a randomly selected tree will be at least 10 in.? Will exceed 10 in.?
b. What is the probability that the diameter of a randomly selected tree will exceed 20 in.?
c. What is the probability that the diameter of a randomly selected tree will be between 5 and 10 in.?
d. What value c is such that the interval $(8.8 - c, 8.8 + c)$ includes 98% of all diameter values?
e. If four trees are independently selected, what is the probability that at least one has a diameter exceeding 10 in.?

36. Spray drift is a constant concern for pesticide applicators and agricultural producers. The inverse relationship between droplet size and drift potential is well known. The paper "Effects of 2,4-D Formulation and Quinclorac on Spray Droplet Size and Deposition" (*Weed Technology*, 2005: 1030–1036) investigated the effects of herbicide formulation on spray atomization. A figure in the paper suggested the normal distribution with mean 1050 μm and standard deviation 150 μm was a reasonable model for droplet size for water (the "control treatment") sprayed through a 760 ml/min nozzle.

a. What is the probability that the size of a single droplet is less than 1500 μm? At least 1000 μm?

b. What is the probability that the size of a single droplet is between 1000 and 1500 μm?

c. How would you characterize the smallest 2% of all droplets?

d. If the sizes of five independently selected droplets are measured, what is the probability that at least one exceeds 1500 μm?

37. Suppose that blood chloride concentration (mmol/L) has a normal distribution with mean 104 and standard deviation 5 (information in the article "Mathematical Model of Chloride Concentration in Human Blood," *J. of Med. Engr. and Tech.*, 2006: 25–30, including a normal probability plot as described in Section 4.6, supports this assumption).

a. What is the probability that chloride concentration equals 105? Is less than 105? Is at most 105?

b. What is the probability that chloride concentration differs from the mean by more than 1 standard deviation? Does this probability depend on the values of μ and σ?

c. How would you characterize the most extreme .1% of chloride concentration values?

38. There are two machines available for cutting corks intended for use in wine bottles. The first produces corks with diameters that are normally distributed with mean 3 cm and standard deviation .1 cm. The second machine produces corks with diameters that have a normal distribution with mean 3.04 cm and standard deviation .02 cm. Acceptable corks have diameters between 2.9 cm and 3.1 cm. Which machine is more likely to produce an acceptable cork?

39. **a.** If a normal distribution has $\mu = 30$ and $\sigma = 5$, what is the 91st percentile of the distribution?

b. What is the 6th percentile of the distribution?

c. The width of a line etched on an integrated circuit chip is normally distributed with mean 3.000 μm and standard deviation .140. What width value separates the widest 10% of all such lines from the other 90%?

40. The article "Monte Carlo Simulation—Tool for Better Understanding of LRFD" (*J. of Structural Engr.*, 1993: 1586–1599) suggests that yield strength (ksi) for A36 grade steel is normally distributed with $\mu = 43$ and $\sigma = 4.5$.

a. What is the probability that yield strength is at most 40? Greater than 60?

b. What yield strength value separates the strongest 75% from the others?

41. The automatic opening device of a military cargo parachute has been designed to open when the parachute is 200 m above the ground. Suppose opening altitude actually has a normal distribution with mean value 200 m and standard deviation 30 m. Equipment damage will occur if the parachute opens at an altitude of less than 100 m. What is the probability that there is equipment damage to the payload of at least one of five independently dropped parachutes?

42. The temperature reading from a thermocouple placed in a constant-temperature medium is normally distributed with mean μ, the actual temperature of the medium, and standard deviation σ. What would the value of σ have to be to ensure that 95% of all readings are within .1° of μ?

43. The distribution of resistance for resistors of a certain type is known to be normal, with 10% of all resistors having a resistance exceeding 10.256 ohms and 5% having a resistance smaller than 9.671 ohms. What are the mean value and standard deviation of the resistance distribution?

44. If bolt thread length is normally distributed, what is the probability that the thread length of a randomly selected bolt is

a. Within 1.5 SDs of its mean value?

b. Farther than 2.5 SDs from its mean value?

c. Between 1 and 2 SDs from its mean value?

45. A machine that produces ball bearings has initially been set so that the true average diameter of the bearings it produces is .500 in. A bearing is acceptable if its diameter is within .004 in. of this target value. Suppose, however, that the setting has changed during the course of production, so that the bearings have normally distributed diameters with mean value .499 in. and standard deviation .002 in. What percentage of the bearings produced will not be acceptable?

46. The Rockwell hardness of a metal is determined by impressing a hardened point into the surface of the metal and then measuring the depth of penetration of the point. Suppose the Rockwell hardness of a particular alloy is normally distributed with mean 70 and standard deviation 3. (Rockwell hardness is measured on a continuous scale.)

a. If a specimen is acceptable only if its hardness is between 67 and 75, what is the probability that a randomly chosen specimen has an acceptable hardness?

b. If the acceptable range of hardness is $(70 - c, 70 + c)$, for what value of c would 95% of all specimens have acceptable hardness?

c. If the acceptable range is as in part (a) and the hardness of each of ten randomly selected specimens is independently determined, what is the expected number of acceptable specimens among the ten?

d. What is the probability that at most eight of ten independently selected specimens have a hardness of less than