HW1 SOLUTIONS   TOT. POINTS = 9

10.3 $\displaystyle\sum_{i=1}^{n} i^2 = \frac{1}{6} n(n+1)(2n+1) \quad \forall n \in \mathbb{N}$

[3 Pts]

PROOF by induction. $P(n): \displaystyle\sum_{i=1}^{n} i^2 = \frac{n}{6}(n+1)(2n+1)$

1 Pt

- $n=1$ $P(1)$ holds
- Assume $P(k)$ holds
- Need to show that $P(k+1)$ holds

$$\sum_{i=1}^{k+1} i^2 = \sum_{i=1}^{k} i^2 + (k+1)^2 = \frac{k}{6}(k+1)(2k+1) + (k+1)^2$$ since $P(k)$ holds

2 Pts

Thus $\displaystyle\sum_{i=1}^{k+1} i^2 = \frac{k+1}{6}\left[k(2k+1) + 6(k+1)\right] = \frac{k+1}{6}(k+1+1)(2(k+1)+1)$ □

10.14 Show $P(n): 9^n - 4^n = 5m \quad \forall n \in \mathbb{N}$, when m is a natural number

[3 Pts]

1 Pt

- $P(1)$ holds since $9-4=5$

2 Pts

- Suppose $9^k - 4^k = 5m$ for some $m \in \mathbb{N}$
- $9^{k+1} - 4^{k+1} = 9\,9^k - 4\,4^k = 5\cdot 9^k + 4\cdot 9^k - 4\cdot 4^k = 5\cdot 9^k + 4(9^k - 4^k)$

$= 5\,9^k + 4\cdot 5m$ since $P(k)$ holds

This shows that $9^{k+1} - 4^{k+1} = 5(9^k + 4m) = 5m_1$, $m_1 \in \mathbb{N}$. □

10.21 Show that $n^2 \le 2^n$ for $n \ge 4$

[3 Pts]

Set $P(n): n^2 \le 2^n$

1 Pt

- $P(4)$ holds since $16 \le 2^4 = 16$
- Assume $P(k)$ holds (for a fixed $k \ge 4$)

2 Pts

- $2^{k+1} = 2\cdot 2^k \ge 2k^2$ since $P(k)$ holds

Thus $2^{k+1} \ge k^2 + k^2 \ge k^2 + \underline{2k+1} = (k+1)^2$ for $k \ge 4$ □

To show that $k^2 \ge 2k+1$ for $k \ge 4$,

notice that $k^2 - 2k - 1 \ge 0$ for $k > 1+\sqrt{2}$.