HW #2 SOLUTIONS

11.4 Suppose $x \geq 0$ and $x \leq \varepsilon$ $\forall \varepsilon > 0$. Prove that $x = 0$

proof Arguing by contradiction, suppose that $x > 0$.
Then there is an $\varepsilon > 0$ s.t. $x - \varepsilon > 0$.
This contradicts the hypothesis that $x \leq \varepsilon$ $\forall \varepsilon > 0$

2 Pts

11.7 Prove by induction the statement

$$P(n): \quad |x_1 + x_2 + \dots + x_n| \leq |x_1| + |x_2| + \dots + |x_n|$$

$P(1)$: $|x_1| \leq |x_1|$ is true

Suppose that $P(k)$ holds. That is $|x_1 + \dots + x_k| \leq |x_1| + \dots + |x_k|$

Need to prove $P(k+1)$. By the triangle inequality we have:

$$|x_1 + x_2 + \dots + x_k + x_{k+1}| = |(x_1 + \dots + x_k) + x_{k+1}|$$

$$\leq |x_1 + \dots + x_k| + |x_{k+1}|$$

Since $P(k)$ holds we have:

$$\hookrightarrow \leq |x_1| + \dots + |x_k| + |x_{k+1}|.$$

Thus $P(k+1)$ holds. This completes the proof.

3 Pts

12.6 (a) Let $S \subset \mathbb{R}$ is nonempty and bounded
Suppose m_1 and m_2 are both supremum of S
~~Since m_1 is the upper bound~~ If $m_1 \neq m_2$ then $m_1 < m_2$ or $m_2 < m_1$
In the first case, m_2 is not a sup, in the second case m_1 is not a sup.
Thus it must be $m_1 = m_2$.

(b) Suppose m, n are both max of S. If $m \neq n$, then $m < n$ or $m > n$.
In the first case, m would not be $\max S$; in the second case,
n would not be $\max S$. Thus, it must be $m = n$

4 Pts (2+2)

12.8 Let $S \subseteq T$, where S, T are bounded and nonempty

Since $\inf T \leq t$ $\forall t \in T$ and $S \subset T$, it follows that
$\inf T \leq s$ $\forall s \in S$. This shows that $\inf T$ is a lower bound of S
and, thus, $\inf T \leq \inf S$. Since $\inf S \leq s$ $\forall s \in S$, then
$\inf S \leq \sup S$. ~~Now~~ $\sup T \geq t$ $\forall t \in T$. Since $S \subseteq T$
then $\sup T \geq s$ $\forall s \in S$. Thus $\sup T \geq \sup S$.

Collecting all inequalities, we have

$$\inf T \leq \inf S \leq \sup S \leq \sup T.$$

4 Pts