

- (e) True: Theorem 13.17(a).
- (f) True: Definition 13.14.
- (g) False: \mathbb{R} is both open and closed.
- (h) False: see Example 13.12.
- (i) True: Corollary 13.11(a).

13.3 Answers in book: (a) \emptyset ; (b) $(0,5)$; (c) \emptyset ; (d) \emptyset ; (e) \emptyset .

13.4 (a) $\{0\} \cup \{1/n : n \in \mathbb{N}\}$ (b) $\{0, 5\}$ (c) $[0, \sqrt{2}]$
 (d) $[\sqrt{2}, \infty)$ (e) $\{2\}$

13.5 (a) neither (b) closed (c) neither
 (d) both (e) closed (f) open

13.6 (a) $\{0\} \cup \{1/n : n \in \mathbb{N}\}$ (b) \mathbb{N} (c) \mathbb{R}
 (d) \emptyset (e) $[9/2, 11/2]$ (f) \mathbb{R}

- 13.7 (a) Let $S = \{1/n : n \in \mathbb{N}\}$. Then $P = S$, and P is not closed.
 (b) Let $S = [0, 1] \cup \{2\}$. Then S is closed, but $\text{cl}(\text{int } S) = [0, 1] \neq S$.
 (c) Let $S = (0, 1) \cup (1, 2)$. Then S is open, but $\text{int}(\text{cl } S) = (0, 2) \neq S$.
 (d) Let $S = (0, 1) \cup (1, 2)$. Then $\text{bd } S = \{0, 1, 2\}$, but $\text{cl } S = [0, 2]$, so $\text{bd}(\text{cl } S) = \{0, 2\} \neq \text{bd } S$.
 (e) Let $S = [0, 1] \cap \mathbb{Q}$. Then $\text{bd } S = [0, 1]$, but $\text{bd}(\text{bd } S) = \{0, 1\}$.
 (f) Let $S = [0, 2]$ and $T = [1, 3]$. Then $\text{bd}(S \cup T) = \{0, 3\}$, but $(\text{bd } S) \cup (\text{bd } T) = \{0, 1, 2, 3\}$.
 (g) Let $S = [0, 2]$ and $T = [1, 3]$. Then $\text{bd}(S \cap T) = \{1, 2\}$, but $(\text{bd } S) \cap (\text{bd } T) = \emptyset$.

13.8 If $x \in \text{int } S$, then there exists a neighborhood of x that is contained in S . If $x \notin \text{int } S$, then every neighborhood of x intersects $\mathbb{R} \setminus S$. There are two possibilities. It may be that some neighborhood of x is contained in $\mathbb{R} \setminus S$. In this case, $x \in \text{int}(\mathbb{R} \setminus S)$. If not, then every neighborhood of x must intersect both S and $\mathbb{R} \setminus S$. This means $x \in \text{bd } S = \text{bd}(\mathbb{R} \setminus S)$.

- 13.9 (a) Let x be an accumulation point of set S , suppose x is not an interior point of S , and let N be an arbitrary neighborhood of x . Since x is not an interior point of S , N cannot be contained in S . That is $N \cap (\mathbb{R} \setminus S) \neq \emptyset$. On the other hand, since x is an accumulation point of S , every deleted neighborhood of x , and hence every neighborhood of x , must contain a point of S . Thus $N \cap S \neq \emptyset$. It follows that x is a boundary point of S .
 (b) Let $x \in \text{bd } S$ and suppose $x \in S$. If x is not an accumulation point of S , then by definition x is an isolated point of S . On the other hand, suppose $x \notin S$. Since $x \in \text{bd } S$, every neighborhood of x must intersect S in a point, but this point cannot be x since $x \notin S$. This means every deleted neighborhood of x must contain a point of S , and x is an accumulation point of S .

13.10 Let x be an isolated point of S , let N be an arbitrary neighborhood of x , and suppose $N \subseteq S$. Since every deleted neighborhood of x will have nonempty intersection with N , this would imply that x is an accumulation point of S , a contradiction. Thus N is not a subset of S and $N \cap (\mathbb{R} \setminus S) \neq \emptyset$. Since $x \in S$ and $x \in N$, we have $N \cap S \neq \emptyset$. Thus x is a boundary point of S .

13.11 through 13.18 are routine. Here are the hints in the book:

13.11 $A \setminus B = A \cap (\mathbb{R} \setminus B)$.

13.13 Look at neighborhoods.

13.15 Suppose that some neighborhood of x contains only a finite number of points of S and then find a smaller deleted neighborhood that misses S completely.