Section 1-1

4.

(a) $n = 4$; (b) $\begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$; (c) $a_{23} - a_{31} = -7 - 6 = -13$.

5. $x + y = (5, 0)$.

6. $x + y = (-1, 3, 6)$.

7. Addition is not possible.

8. $A + B = \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix}$.

9. Addition is not possible.

10. $4A + B = \begin{pmatrix} 8 & 6 \\ -1 & 15 \end{pmatrix}$.

Section 1-2

2. Typing `x + y` should generate a MATLAB error in both cases.

3. $3.27x - 7.4y = (23.1640, -3.5620, -12.8215)$.

5. $-4.2A + 3.1B = \begin{pmatrix} -14.0300 & -5.8470 & 7.0600 \\ -9.7600 & 11.0570 & -9.6600 \end{pmatrix}$.

Section 1-3

1. The matrix is symmetric.

2. The matrix is not symmetric.

3. The matrix is symmetric.

4. The matrix is not symmetric.

5. The matrix is symmetric.

7. The matrix is strictly upper triangular.

9. The matrix is not upper triangular since a triangular matrix must be square.

10. The matrix is upper triangular.

11. 3.

12. 3.

15. $1 + 2 + \cdots + (n-1) + n = \frac{n(n+1)}{2}$.

17. *True.*

18. *False* — for example: $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$.

19. *False* — for example: $\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$.

Section 1-4

2. Answer: The length of x is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$.

3. Answer: The length of x is $\sqrt{(-1)^2 + 1^2 + 1^2} = \sqrt{3}$.

6. Answer: The vectors are not perpendicular.

SOLUTION Compute: $(2, -1) \cdot (-2, 1) = -5$.

7. Answer: The vectors are not perpendicular.

SOLUTION Compute: $(1, 1, 3, 5) \cdot (1, -4, 3, 0) = 6$.

8. Answer: The vectors are perpendicular.

SOLUTION Compute: $(2, 1, 4, 5) \cdot (1, -4, 3, -2) = 0$.

9. When $a = \frac{10}{3}$, x and y are perpendicular, since $(1, 3, 2) \cdot (2, a, -6) = 3a - 10 = 0$.

10. $||u|| = \sqrt{2^2 + 1^2 + (-2)^2} = 3$;

$||v|| = \sqrt{0^2 + 1^2 + (-1)^2} = \sqrt{2}$;

$\cos\theta = \dfrac{u \cdot v}{||u||\,||v||} = \dfrac{3}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4} = 45^\circ$.

13. Answer: The dot product $x \cdot y = 13$, and the cosine of the angle θ between x and y is $\frac{13}{6\sqrt{5}}$.

SOLUTION Compute $x \cdot y = 13$, $||x|| = 3\sqrt{2}$, and $||y|| = \sqrt{10}$. Then by Theorem 1.4.1,

$$\cos\theta = \frac{x \cdot y}{||x||\,||y||} = \frac{13}{3\sqrt{20}} = \frac{13}{6\sqrt{5}}.$$

15. Answer: The dot product $x \cdot y = 31$, and the cosine of the angle θ between x and y is

$$\frac{31}{\sqrt{1410}} \approx 0.8256.$$

SOLUTION Compute $x \cdot y = 31$, $||x|| = \sqrt{15}$, and $||y|| = \sqrt{94}$. Then by Theorem 1.4.1,

$$\cos\theta = \frac{x \cdot y}{||x||\,||y||} = \frac{31}{\sqrt{15}\sqrt{94}} = \frac{31}{\sqrt{1410}} \approx 0.8256.$$

19. $\dfrac{x}{||x||} = (0.1244, 0.8397, -0.4167, 0.3253)$.

21. $\theta = \arccos\left(\dfrac{x \cdot y}{||x||\,||y||}\right) = 0.9634 = 15.5570°$.