Solutions, Assignment #4

Section 3.5

2. AB is not defined. $BA = \begin{pmatrix} 8 & 20 & 0 \\ -4 & -16 & 3 \end{pmatrix}$

3. AB is not defined. BA is not defined.

4. AB is not defined. BA is not defined.

6. $\begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & -1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -2 & 5 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 2-4+3 & 3+10-3 \\ -4-6-1 & -6+15+1 \end{pmatrix}$

$= \begin{pmatrix} 1 & 10 \\ -11 & 10 \end{pmatrix}.$

8. $\begin{pmatrix} 2 & -1 & 3 \\ 1 & 0 & 5 \\ 1 & 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 7 \\ -2 & -1 \\ -5 & 3 \end{pmatrix} = \begin{pmatrix} 2+2-15 & 14+1+9 \\ 1-25 & 7+15 \\ 1-10+5 & 7-5-3 \end{pmatrix}$

$= \begin{pmatrix} -11 & 24 \\ -24 & 22 \\ -4 & -1 \end{pmatrix}.$

9. **Answer:** For any matrix B of the form

$$\begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix}$$

the equation $AB = BA$ is valid.

SOLUTION Let

$$B = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}.$$

Then compute the matrix B for which

$$\begin{aligned} AB &= BA \\ \begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} &= \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix} \\ \begin{pmatrix} 2b_{11} & 2b_{12} \\ -b_{21} & -b_{22} \end{pmatrix} &= \begin{pmatrix} 2b_{11} & -b_{12} \\ 2b_{21} & -b_{22} \end{pmatrix} \end{aligned}$$

which is equivalent to the linear system

$$\begin{aligned} 2b_{11} &= 2b_{11} \\ 2b_{12} &= -b_{12} \\ -b_{21} &= 2b_{21} \\ -b_{22} &= -b_{22}. \end{aligned}$$

13. The matrix AB is not defined because A has 5 columns while B has four rows. The matrix BA is also not defined because B has 6 columns and A has 3 rows.

14. Both AB and BA are defined and can be computed using MATLAB:

```
A*B

ans =

   -34   -26     7   -23
    -6    19    25    15
   -15     6    -5     7
    -4   -11    12    -6
```

```
B*A

ans =

    -8     4    -8    35
    -8    -1    28     3
     2    27     0   -43
     7     0     3   -17
```

Section 3.6

4. (a) Verify $J^2 = -I$ by computation:

$$J^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -I.$$

(b) **Answer:** $(aI + bJ)(cI + dJ) = (ac - bd)I + (ad + bc)J$.

SOLUTION Evaluate $(aI + bJ)(cI + dJ)$, yielding $acI^2 + adIJ + bcJI + bdJ^2$. Then, use the identities $IJ = JI = J$, $I^2 = I$, and $J^2 = -I$ to rewrite the expression in terms of I and J.

8. Conmputer experiment.

Section 3.7

2. We can compute

$$(\alpha A)\left(\frac{1}{\alpha}A^{-1}\right) = \left(\alpha\frac{1}{\alpha}\right)(AA^{-1}) = I.$$

So the inverse of αA is indeed $\frac{1}{\alpha}A^{-1}$.

5. **Answer:** $A^{-1} = \dfrac{1}{10}\begin{pmatrix} -8 & 32 & -9 \\ 2 & 2 & 1 \\ 2 & -8 & 1 \end{pmatrix}$.

SOLUTION Let

$$M = (A|I_3) = \left(\begin{array}{rrr|rrr} 1 & 4 & 5 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -2 & 0 & -8 & 0 & 0 & 1 \end{array}\right).$$

Then, row reduce M to obtain the augmented matrix $(I_3|A^{-1})$.

6. Answer: $B^{-1} = \begin{pmatrix} -1 & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & 0 \\ -2 & -1 & 1 \end{pmatrix}$.

SOLUTION Let

$$N = (B|I_3) = \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 2 & 0 & -1 & 0 & 0 & 1 \end{array}\right).$$

Row reduce N to obtain the augmented matrix $(I_3|B^{-1})$.

10. Type `M = [A eye(3)]` in MATLAB, then row reduce the augmented matrix `M`, obtaining:

```
ans =
    1.0000         0         0    0.1667   -0.1667    0.1667
         0    1.0000         0   -0.8333    0.8333    0.1667
         0         0    1.0000    0.5000   -0.1667   -0.1667
```

Check your answer using `inv(A)` which confirms

```
ans =
    0.1667   -0.1667    0.1667
   -0.8333    0.8333    0.1667
    0.5000   -0.1667   -0.1667
```

11. Type `N = [B eye(4)]` in MATLAB, then row reduce `N` to obtain:

```
ans =
     1   0         0         0   -1.5714   -0.4286         0    1.4286
     0   1         0         0    0.7429    0.0571    0.2000   -0.4571
     0   0         1         0   -0.9143    0.3143   -0.4000    0.4857
     0   0         0         1   -0.6000   -0.2000   -0.2000    0.6000
```

The command `inv(B)` returns the right half of this augmented matrix:

```
ans =
   -1.5714   -0.4286         0    1.4286
    0.7429    0.0571    0.2000   -0.4571
   -0.9143    0.3143   -0.4000    0.4857
   -0.6000   -0.2000   -0.2000    0.6000
```

Section 3.8

1. Use (3.8.1) to compute the inverse of the matrix, as follows:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}^{-1} = \frac{1}{4-3}\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}.$$

2. Answer: The inverse of the matrix is

$$\begin{pmatrix} 1 & -K \\ 0 & 1 \end{pmatrix}.$$

SOLUTION Note that the inverse of any 2×2 matrix is:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

3. Case: $a \neq 0$. A can be row reduced as follows:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ c & d \end{pmatrix} \to \begin{pmatrix} 1 & \frac{b}{a} \\ 0 & \frac{ad-bc}{a} \end{pmatrix}.$$

If $ad-bc \neq 0$, then the matrix can be row reduced to I_2, whereas if $ad-bc=0$, the row reduced matrix is:

$$\begin{pmatrix} 1 & \frac{b}{a} \\ 0 & 0 \end{pmatrix}$$

which cannot be reduced further and is not row equivalent to I_2.

Case: $a=0$. If either $c=0$ or $b=0$, then the resulting matrices,

$$\begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix}$$

respectively, are not row equivalent to I_2, and $ad-bc = 0-0 = 0$. If $c \neq 0$ and $b \neq 0$, then the matrix can be row reduced:

$$\begin{pmatrix} 0 & b \\ c & d \end{pmatrix} \to \begin{pmatrix} c & d \\ 0 & b \end{pmatrix} \to \begin{pmatrix} 1 & \frac{d}{c} \\ 0 & b \end{pmatrix}$$

which is row equivalent to I_2. So A is indeed row equivalent to I_2 if and only if $ad-bc \neq 0$.

6. Answer: Let T be the triangle whose vertices are 0, p, and q, and let U be the triangle whose vertices are 0, Mp and Mq. Then $A_T = 3$ is the area of T, and $A_U = 69$ is the area of U.

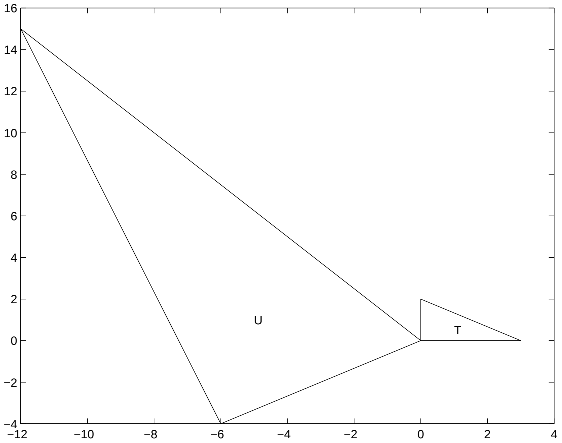

Figure 6

SOLUTION Use the formula for the area of a triangle to compute $A_T = \frac{1}{2}(3)(2) = 3$. Then, use Theorem 3.8.5 to compute

$$A_U = |\det M| A_T = 23(3) = 69.$$

Figure 6 shows triangles T and U.

8. Answer: $x = \frac{13}{19}$.

SOLUTION By Cramer's rule (see (**??**)),

$$x = \det \begin{pmatrix} 2 & 3 \\ 1 & -5 \end{pmatrix} \Big/ \det \begin{pmatrix} 2 & 3 \\ 3 & -5 \end{pmatrix} = \frac{-13}{-19}\ .$$

12. Answer: The matrix A is not invertible and $\det(A) = 0$.

SOLUTION Figure 12 shows the `map` output for this matrix. The square is mapped to a line, whose area is 0, so $|\det(A)| = 0$.

13. Answer: The matrix A is not invertible and $\det(A) = 0$.

SOLUTION Figure 13 shows the `map` output for this matrix. The square is mapped to a line, whose area is 0, so $|\det(A)| = 0$.

14. Answer: The matrix A is invertible and $\det(A) = 1$.

SOLUTION Figure 14 shows the `map` output for this matrix. The result of the map is another square of area 1, so $|\det(A)| = 1$.

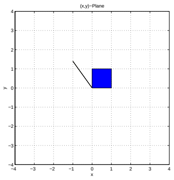

Figure 12

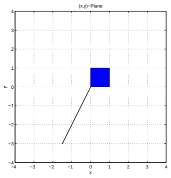

Figure 13

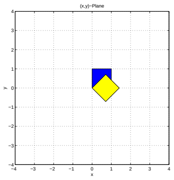

Figure 14