Solutions, Assignment #5

Section 4.1

1. Answer: The determinant of the matrix is -28.

SOLUTION Expand along the third column, obtaining:

$$\det\begin{pmatrix} -2 & 1 & 0 \\ 4 & 5 & 0 \\ 1 & 0 & 2 \end{pmatrix} = 2\det\begin{pmatrix} -2 & 1 \\ 4 & 5 \end{pmatrix} = 2(-14) = -28.$$

2. Answer: The determinant of the matrix is -110.

SOLUTION First row reduce:

$$\det\begin{pmatrix} 1 & 0 & 2 & 3 \\ -1 & -2 & 3 & 2 \\ 4 & -2 & 0 & 3 \\ 1 & 2 & 0 & -3 \end{pmatrix} = \det\begin{pmatrix} 1 & 0 & 2 & 3 \\ 0 & -2 & 5 & 5 \\ 0 & -2 & -8 & -9 \\ 0 & 2 & -2 & -6 \end{pmatrix}.$$

Then, use formula (**??**):

$$\begin{aligned} \det\begin{pmatrix} -2 & 5 & 5 \\ -2 & -8 & -9 \\ 2 & -2 & -6 \end{pmatrix} &= \begin{array}{l} (-2)(-8)(-6) + 5(-9)2 + 5(-2)(-2) - 5(-8)2 \\ -5(-2)(-6) - (-2)(-9)(-2) \end{array} \\ &= -96 - 90 + 20 + 80 - 60 + 36 \\ &= -110. \end{aligned}$$

4. Answer: The solution is $\det(A^{-1}) = \frac{1}{35}$.

SOLUTION By Definition **??**(c), $\det(A)\det(A^{-1}) = \det(I_3) = 1$. Therefore,

$$\det(A^{-1}) = \frac{1}{\det(A)}.$$

Now compute $\det(A)$ using (**??**):

$$\det(A) = -2\begin{pmatrix} 1 & 3 \\ 1 & 1 \end{pmatrix} + 3\begin{pmatrix} 4 & 3 \\ -1 & 1 \end{pmatrix} + 2\begin{pmatrix} 4 & 1 \\ -1 & 1 \end{pmatrix} = 35.$$

6. Answer: The determinant is $\det(A) = 18$.

SOLUTION Compute by row reduction as follows:

$$\begin{aligned}
\det\begin{pmatrix} -1 & -2 & 1 \\ 3 & 1 & 3 \\ -1 & 1 & 1 \end{pmatrix} &= -\det\begin{pmatrix} 1 & 2 & -1 \\ 0 & -5 & 6 \\ 0 & 3 & 0 \end{pmatrix} \\
&= 3\det\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & -5 & 6 \end{pmatrix} \\
&= 3\det\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 6 \end{pmatrix} = 18.
\end{aligned}$$

8. Answer: The determinant is $\det(C) = -7$.

SOLUTION Compute by row reduction:

$$\det\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 1 & 0 \\ -2 & -3 & 3 & -1 \\ 1 & 0 & 5 & 2 \end{pmatrix} = 5\det\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & \frac{2}{5} \\ 0 & 0 & 0 & -\frac{7}{5} \end{pmatrix} = 5\left(-\frac{7}{5}\right) = -7.$$

14. Answer: The determinant of A_λ vanishes at $\lambda = -1$, $\lambda = 1$, and $\lambda = 2$.

SOLUTION Compute $\det(A_\lambda)$ by expanding along the first column to obtain:

$$\begin{aligned}
\det(A_\lambda) &= (\lambda-1)((\lambda-1)\lambda-1) - 1(\lambda-1) \\
&= (\lambda-1)(\lambda-2)(\lambda+1).
\end{aligned}$$

Section 4.2

3. $\det A = -2$; A^{-1} exists; $A^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

5. $\det A = 1$; A^{-1} exists; $A^{-1} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$

6. $\det A = 2$; A^{-1} exists; $A^{-1} = \begin{pmatrix} -16 & -11 & 3 \\ 7/2 & 5/2 & -1/2 \\ -5/2 & -3/2 & 1/2 \end{pmatrix}$

7. $\det A = 0$; A^{-1} does not exist.

17. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1/9 & 1/3 & 5/9 \\ 1/3 & 0 & -1/3 \\ -2/9 & 1/3 & -1/9 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 7/9 \\ 1/3 \\ -5/9 \end{pmatrix}$

20. 0

21. -45

26. -10

27. -18

28. -124

32. $\det A = -3$; Cramer's rule applies; $x = \dfrac{2}{3}$.

33. $\det A = -37$; Cramer's rule applies; $y = -\dfrac{25}{37}$.