MATH 1431 **EXAM 2 REVIEW - ANSWERS**

Part I. Differentials; Newton-Raphson Approximations

1. $f(5.4) \cong 2.2$

2. (i) $(62)^{2/3} \cong \dfrac{47}{3} \cong 15.667$ (ii) $\cos 33^o \cong 0.8395$

3. approximately 979 gallons

4. $x_2 = \dfrac{79}{27} \approx 2.9259;$ $x_3 = \dfrac{79}{27} - \dfrac{\left(\frac{79}{27}\right)^3 - 25}{3\left(\frac{79}{27}\right)^2} \approx 2.9240$

Part II. The Mean-Value Theorem; Rolle's theorem

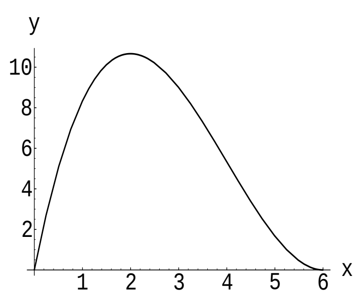
1. Set $f(x) = x^3 + 4x - 1$. Since $f(x) \to -\infty$ as $x \to -\infty$ and $f(x) \to \infty$ as $x \to \infty$, f has at least one zero (the equation has at least one real root). Since $f'(x) = 3x^2 + 4 > 0$, f has exactly one zero (the equation has exactly one root).

2. No. By the mean-value theorem there exists at least one number $c \in (-2, 2)$ such that

$$f'(c) = \frac{f(2) - f(-1)}{2 - (-1)} = -\frac{4}{3} < 0$$

3. $c = \pm\dfrac{2}{\sqrt{3}}$

4.



Part III. Local and Absolute Extrema

1. f has local minima at $c = -1$ and $c = 2$; f has a local maximum at $c = 0$.

2. $g(-3) = 12$ is the absolute maximum; $g(-2) = -13$ is the absolute minimum

3. $f(-2) = 1$ endpoint and absolute min; $f(-1) = 2$ local max; $f(-1/3)$ loca min; $f(1)$ endpoint and absolute max.

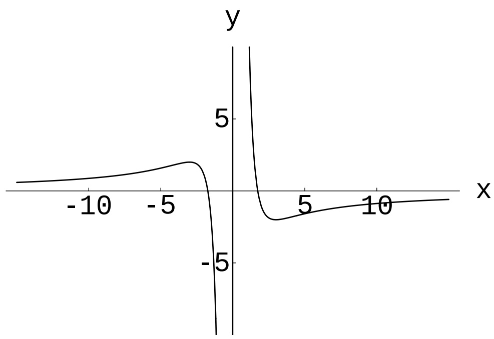
Part IV. Curve Sketching

1. domain: all real numbers except $x = 0$; no y-intercepts, x-intercepts: $x = \pm\sqrt{3}$; symmetric with respect to the origin (f is an odd function); vertical asymptote: $x = 0$, horizontal asymptote: $y = 0$

2. critical numbers: $x = \pm 3$; f is increasing on $(-\infty, 3]$ and on $[3, \infty)$; f is decreasing on $[3, 0)$ and on $(0, 3]$

3. The graph of f is concave down on $(-3\sqrt{2}, 0)$ and on $(3\sqrt{2}, \infty)$; The graph of f is concave up on $(-\infty, 2\sqrt{3})$ and on $(0, 3\sqrt{2})$; points of inflection at $x = \pm 3\sqrt{2}$

4.

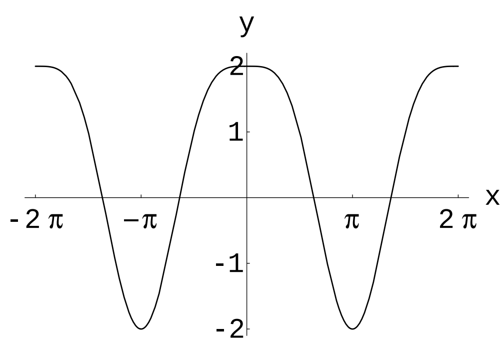


5. symmetric with respect to the y-axis (f is an even function)

6. critical numbers: $x = -\pi,\ 0,\ \pi$; f is increasing on $(-\pi, 0]$ and on $[\pi, 2\pi]$; f is decreasing on $[-2\pi, -\pi]$ and on $[0, \pi]$

7. The graph of f is concave down on $\left(-2\pi, -\frac{4}{3}\pi\right)$, $\left(-\frac{2}{3}\pi, \frac{2}{3}\pi\right)$ and $\left(\frac{4}{3}\pi, 2\pi\right)$; The graph of f is concave up on $\left(-\frac{4}{3}\pi, -\frac{2}{3}\pi\right)$ and $\left(\frac{2}{3}\pi, \frac{4}{3}\pi\right)$; points of inflection at $x = -\frac{4}{3}\pi,\ -\frac{2}{3}\pi,\ \frac{2}{3}\pi,\ \frac{4}{3}\pi$

8.



Part V. Max-Min Problems

1. (i) $C(x) = 300x + \dfrac{480(4000)}{x}$; domain: $(0, \infty)$

(ii) dimensions for minimum cost: length 80 feet, width 50 feet; minimum cost: $C(80) = \$48,000$

2. (i) $V(x) = 108x^2 - 4x^3$; domain: $[0, 27]$

(ii) dimensions for max. volume: $18 \times 18 \times 36$ (in inches); max. volume: $V(18) = 11,664$ cu. in