Answers to Odd-Numbered Problems

CHAPTER 1

Exercises 1.1

1. (a) ordinary, first order

(c) partial, second order

(e) ordinary, third order

(g) ordinary, second order

3. Both y and z are solutions.

5. Both y and z are solutions.

7. Both u_1 and u_2 are solutions.

9. u_1 is a solution; u_2 is not a solution.

11. $y = 16x^2 + C_1x + C_2$

13. $y = Ce^{3x}$.

15. $r = 2,\ -2;$ $y_1(x) = e^{2x}$ and $y_2(x) = e^{-2x}$ are solutions.

17. $r = 3;$ $y(x) = e^{3x}$ is a solution.

19. No real values of r; $r = 1 \pm 2i$ are complex values.

21. $r = 3,\ -3;$ $y_1(x) = x^3$ and $y_2(x) = x^{-3}$ are solutions.

Exercises 1.3

1. (b) $y = 2e^{5x}$.

3. (b) $y = \dfrac{e}{e - 2e^x}$.

5. (b) $y = -\sin 3x + \frac{1}{3}\cos 3x$.

7. (b) $y = -\frac{17}{4}x + 9\sqrt{x}$.

(c) y' is not defined at $x = 0$; there is no solution to $y'(0) = 2$.

11. $xy' - 3y + 3 = 0$.

13. $y' - 2y = -4e^{-2x}$.

15. $y'^2 + xy' - y = 0$.

17. $y'' - 4y' + 4y = 0$.

19. $x^2y'' + xy' - y = 0.$

21. $y'' + 9y = 0.$

23. $y''' = 0.$

CHAPTER 2

Exercises 2.1

1. $y = -\frac{1}{2} + Ce^{2x}.$

3. $y = 1 + Ce^{-x^2}.$

5. $y = e^{-x} + Ce^{x}.$

7. $y = x^{-2}\sin x + Cx^{-2}.$

9. $y = \frac{2}{9}(x+1)^{5/2} + C(x+1)^{-2}.$

11. $y = \sin x \cos x + C\cos x = \frac{1}{2}\sin 2x + C\cos x.$

13. $y = e^x + \dfrac{C}{x}.$

15. $y = x(\ln x)^2 + Cx.$

17. $y = 1 + Ce^{-e^{x}}.$

19. $y = x - 1 + 2e^{-x}.$

21. $y = \dfrac{\ln(1+e^x)}{e^x} + (e - \ln 2)e^{-x}.$

23. $y = \dfrac{5 - \cos 2x}{2\sin x}.$

25. $y = \dfrac{2}{Cx - 3x^3}.$

27. $y = \left(Ce^{2x} - e^{x}\right)^2.$

29. $y = \dfrac{1}{\sqrt[3]{Cx^3 - 2x^3\ln x}}.$

Exercises 2.2

1. $y = \left(\dfrac{x^2}{4} + C\right)^2.$

3. $\tan^{-1} y = x^3 + C$ or $y = \tan\left(x^3 + C\right).$

5. $\cot y = \ln\sqrt{\dfrac{1-x}{1+x}} + C.$

7. $e^{-y} = e^x - xe^x + C.$

9. $y = \dfrac{x+C}{1-Cx}.$

11. $y^2 = C(\ln x)^2 - 1.$

13. $\ln|y| = -\ln|x| - \dfrac{1}{x} - 1.$

15. $y = xe^{x^2-1}.$

17. $y + \ln|y| = \frac{1}{3}x^3 - x - 5.$

19. $y^2 = \dfrac{C}{1+x^2} - 1.$

21. $y = \dfrac{\ln|\sec x + \tan x|}{x} + \dfrac{C}{x}.$

23. $y = 1 + Ce^{-x^2}.$

25. $y = C(3x^2+1)^{1/3} - 3.$

27. $y = \dfrac{1}{Cx + 1 + \ln x}.$

29. $y = \sqrt{x^2 + Cx}.$

31. $x\ln x + \dfrac{x+y}{e^{y/x}} = Cx.$

33. $\csc(y/x) - \cot(y/x) = Cx.$

Exercises 2.3.1

1. $x^2 + 3y^2 = C.$

3. $\dfrac{x^2}{2} + y^2 - 4y = C.$

5. $\dfrac{x^2}{2} + y^2 = C;$ ellipses, center at the origin, major axis horizontal.

7. $y = C(x-a)$

Exercises 2.3.2

1. (a) $A(t) = 50\left(\dfrac{9}{10}\right)^{t/2} \approx 50e^{-0.05268t}.$ (b) $A(4) = 50\left(\frac{9}{10}\right)^2 = 40.5$ grams.

(c) $T \approx 13.16$ hours.

3. (a) $P(t) \approx 0.25e^{0.0421t}$ (b) ≈ 1.6573 square centimeters (c) ≈ 16.464 hours

5. (a) $P(t) \approx 4.5e^{0.01438t}.$ (b) 48.19 years (c) ≈ 6.17 billion.

Exercises 2.3.3

1. (a) $40.1°$. (b) 1.62 minutes.

3. 8:52 pm.

Exercises 2.3.4

1. (a) $v = \left(v_0 + \frac{g}{r}\right) e^{-rt} - \frac{g}{r}$ (b) $\lim_{t\to\infty} v = -\frac{g}{r}$.

(c) $y = y_0 + \frac{1}{r}\left(v_0 + \frac{g}{r}\right)\left(1 - e^{-rt}\right) - \frac{g}{r}t$

3. $k \approx 17.8$

Exercises 2.3.5

1. (a) $A(t) = 10,000\left(1 - e^{-t/200}\right)$ (b) $t = 200\ \ln 5 \approx 322$ minutes

3. (a) $A(t) = \frac{9}{2}\left(1 - e^{-t/150}\right)$ (b) $t = 150\ \ln 3 \approx 165$ minutes

5. (a) $A(t) = \frac{3}{20}\,t(100 - t)$ (b) $\max = A(50) = 375$

Exercises 2.3.6

1. (a) 3259 people. (b) ≈ 6.89 days.

CHAPTER 3

Exercises 3.2

1. Yes

3. Yes

5. Yes

7. (a) $r = -1,\ r = 4$.

(b) Fundamental set: $y_1(x) = x^{-1},\ y_2(x) = x^4$; general solution: $y = C_1 x^{-1} + C_2 x^4$.

(c) $y = \frac{9}{5}\, x^{-1} + \frac{1}{5}\, x^4$.

(d) The trivial solution: $y \equiv 0$.

9. $y'' - 2y' - 3y = 0$.

11. $y'' = 0$.

13. $x^2\, y'' - 2x\, y' + 2\, y = 0$.

15. $W[y_1, y_2](x) = e^{-\int_a^x p(t)\, dt} \neq 0$ for all x.

17. $\{y_1(x) = x,\ y_2(x) = x^2\}$.

19. $\{y_1(x) = e^{x^2},\ y_2(x) = e^{-x^2}\}$.

21. $\alpha\delta - \beta\gamma \neq 0$.

23. $W[y_1 + y_2, y_1 - y_2] = -2W[y_1, y_2]$.

25. Set $u(x) = \dfrac{y_2(x)}{y_1(x)}$. Then

$$u'(x) = \frac{y_1 y_2' - y_2 y_1'}{y_1^2} = \frac{W[y_1, y_2]}{y_1^2} \equiv 0.$$

Therefore, $u \equiv \lambda$ constant, which implies that $y_2 = \lambda\, y_1$.

Exercises 3.3

1. $y = C_1 e^{2x} + C_2 e^{-4x}$.

3. $y = C_1 e^{5x} + C_2\, x\, e^{5x}$.

5. $y = e^{-2x}\, [C_1 \cos 3x + C_2 \sin 3x]$.

7. $y = C_1 + C_2 e^{-2x}$.

9. $y = C_1 e^{2\sqrt{3}\, x} + C_2 e^{-2\sqrt{3}\, x}$.

11. $y = e^{x}\, [C_1 \cos x + C_2 \sin x]$.

13. $y = C_1 e^{6x} + C_2 e^{-5x}$.

15. $y = e^{-x/2}\left[C_1 \cos x/2 + C_2 \sin x/2\right]$.

17. $y = C_1 e^{4x} + C_2\, x\, e^{4x}$.

19. $y = 2e^{2x} - e^{3x}$.

21. $y = -3e^{-x} - 2xe^{-x}$.

23. $y = -e^{x} \cos x$.

25. $y'' + 3y' - 10y = 0$.

27. $y'' + 4y = 0$.

29. $y'' - \frac{5}{2}\, y' + y = 0$.

31. $y'' + 2y' + 10y = 0$.

33. $y'' + 16y = 0$.

35. $y = (1+\beta)e^{x/2} + (1-\beta)e^{-x/2};\ \beta = -1$.

37. If the roots of $r^2 + ar + b = 0$ are real (real and unequal, or real and equal), then they are negative; r negative implies $e^{rx} \to 0$ and $xe^{rx} \to 0$ as $x \to \infty$. If the roots are complex conjugates, then they have negative real part and α negative implies $e^{\alpha x} \cos \beta x \to 0$ and $e^{\alpha x} \sin \beta x \to 0$ as $x \to \infty$.

39. Suppose that $a > 0$ and $b = 0$. Then the general solution of the differential equation is

$$y = C_1 + C_2 e^{-ax} \qquad \text{and} \qquad \lim_{x\to\infty} y = C_1.$$

The solution that satisfies the initial conditions is: $y = \left(\alpha + \dfrac{\beta}{a}\right) - \dfrac{\beta}{a}\, e^{-ax}; \quad k = \alpha + \dfrac{\beta}{a}$.

41. $$r_1,\ r_2 = \frac{-a \pm \sqrt{a^2 - 4b}}{2} = \frac{-a}{2} \pm \frac{\sqrt{a^2 - 4b}}{2} = \alpha \pm \beta.$$

General solution:

$$\begin{aligned} y = C_1\, e^{(\alpha+\beta)\,x} + C_2\, e^{(\alpha-\beta)\,x} &= C_1\, e^{\alpha\,x}\, e^{\beta\,x} + C_2\, e^{\alpha\,x}\, e^{-\beta\,x} \\ &= e^{\alpha\,x}\left[(C_1 + C_2)\frac{e^{\beta\,x} + e^{-\beta\,x}}{2} + (C_1 - C_2)\frac{e^{\beta\,x} - e^{-\beta\,x}}{2}\right] \\ &= e^{\alpha\,x}\left(K_1 \cosh \beta x + K_2 \sinh gbx\right). \end{aligned}$$

43. $y = C_1\, x^{-2} + C_2\, x^4$.

45. $y = C_1\, x^2 + C_2\, x^2 \ln x$.

Exercises 3.4

1. $z(x) = x^2 \ln x + \frac{1}{2}$; $y = C_1 x^2 + C_2 x^{-1} + x^2 \ln x + \frac{1}{2}$.

3. $z(x) = -x^2 \ln x + \frac{1}{2} x^2 (\ln x)^2$; $y = C_1 x + C_2 x^2 - x^2 \ln x + \frac{1}{2} x^2 (\ln x)^2$.

5. $z(x) = -(1 + x^2)$; $y = C_1 x + C_2 e^x - (1 + x^2)$.

7. $y = C_1 e^{-x} + C_2 e^{2x} - \frac{2}{3} x e^{-x}$.

9. $y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{4} \cos 2x \ln(\cos 2x) + \frac{1}{2} x \sin 2x$.

11. $y = C_1 e^x + C_2 x e^x - e^x \cos x$.

13. $y = C_1 e^{-2x} + C_2 x e^{-2x} - e^{-2x} \ln x$.

15. $y = C_1 \cos 3x + C_2 \sin 3x + \sin 3x \ln(\sec 3x + \tan 3x) - 1$.

17. $y = C_1 x + C_2 x^{-1} + x \ln x$.

19. $y = C_1 x + C_2 x \ln x + x^2$.

Exercises 3.5

1. $y = C_1 e^{-x} + C_2 e^{3x} - e^{2x}$.

3. $y = C_1 e^{-3x} + C_2 x e^{-3x} + \frac{1}{4} e^{3x}$.

5. $y = C_1 e^{-2x} + C_2 - \frac{1}{2} \cos 2x - \frac{1}{2} \sin 2x$.

7. $y = C_1 e^{-x/2} + C_2 e^{-x} + x^2 - 6x + 14 - \frac{9}{10} \cos x - \frac{3}{10} \sin x$.

9. $y = C_1 e^{-2x} + C_2 e^{-3x} + \frac{1}{2} x + \frac{1}{4}$.

11. $y = C_1 e^{-2x} + C_2 e^{-4x} + \frac{3}{2} x e^{-2x}$.

13. $y = C_1 \cos 3x + C_2 \sin 3x + \frac{2}{3} + \frac{1}{162} \left(9x^2 - 6x + 1\right) e^{3x}$.

15. $y = e^x (C_1 \cos 2x + C_2 \sin 2x) - \frac{1}{10} e^{-x} \cos 2x - \frac{1}{20} e^{-x} \sin 2x$.

17. $y = e^x - \frac{1}{2} e^{-2x} - x - \frac{1}{2}$.

19. $y = \frac{13}{15} e^{-x} + \frac{1}{12} e^{2x} + \frac{1}{20} \cos 2x - \frac{3}{20} \sin 2x$.

21. $z = A + (Bx^2 + Cx) e^{-x} + D \cos 3x + E \sin 3x$.

23. $z = Ax^2 + Bx + C + Dx \cos x + Ex \sin x$.

25. $z = (Ax^3 + Bx^2) e^{2x} + Cx^2 + Dx + E + (Fx + G) \cos 2x + (Hx + I) \sin 2x$.

27. $z = Ae^{-x} + Bxe^{-x} \cos x + Cxe^{-x} \sin x + D$.

29. $y = C_1 e^{2x} + C_2 x e^{2x} + \frac{8}{25} \cos x + \frac{6}{25} \sin x + 3x e^{2x} \ln x$.

31. $y = C_1 \cos 3x + C_2 \sin 3x + \frac{3}{8} \cos x - \sin 3x \ln(\sec 3x + \tan 3x) + 1.$

33. $y_1 - y_2$ is a solution of the reduced equation $y'' + ay' + by = 0$ with $a,\ b > 0$. As shown in Exercises 3.4, Problem 37, $y_1 - y_2 \to 0$ as $x \to \infty$. If $a = 0,\ b > 0$, then all solutions of the reduced equation are bounded (Problem 38, Exercises 3.4).

Exercises 3.6

1. The equation of motion is $y(t) = \sin\left(8t + \dfrac{1}{2}\pi\right)$. The amplitude is 1 and the frequency is $8/2\pi = 4/\pi$.

3. The velocity at the equilibrium point is: $\pm 2\pi A/T$.

5. (a) $A\sin(\omega t + \phi_0) = A\cos(\omega t + \phi_0 - \frac{\pi}{2})$; take $\phi_1 = \phi_0 - \frac{1}{2}\pi$.

(b) $A\sin(\omega t + \phi_0) = A\cos\phi_0 \sin\omega t + A\sin\phi_0\cos\omega t = B\sin\omega t + C\cos\omega t$.

7. Assume that $r_1 > r_2$. If $C_1 = 0$ or $C_2 = 0$, then $y = C_1 e^{r_1 t} + C_2 e^{r_2 t}$ can never be zero. If both C_1 and C_2 are nonzero, then $C_1 e^{r_1 t} + C_2 e^{r_2 t} = 0$ implies $e^{(r_1 - r_2)t} = -\dfrac{C_2}{C_1}$. Since $e^{(r_1 - r_2)t}$ is an increasing function $(r_1 > r_2)$, it can take the value $\dfrac{C_2}{C_1}$ at most once. By the same reasoning, $x'(t) = C_1 r_1 e^{r_1 t} + C_2 r_2 e^{r_2 t}$ can be zero at most once. Therefore the motion can change direction at most once.

9. If $\gamma \neq \omega$, we try $z = A\cos\gamma t + B\sin\gamma t$ as a particular solution of $y'' + \omega^2 y = \dfrac{F_0}{m}\cos\gamma t$. Substituting z into the equation, we get $-\gamma^2 z + \omega^2 z = \frac{F_0}{m}\cos\gamma t$, giving

$$z = \frac{F_0/m}{\omega^2 - \gamma^2}\cos\gamma t.$$

11. If $\gamma = \omega$, we try $z = At\cos\omega t + Bt\sin\omega t$ as a particular solution of

$$y'' + \omega^2 y = \frac{F_0}{m}\cos\omega t.$$

Substituting z into the equation, we have

$$(2B\omega - A\omega^2 t)\cos\omega t - (2A\omega + B\omega^2 t)\sin\omega t + \omega^2(At\cos\omega t + Bt\sin\omega t) = \frac{F_0}{m}\cos\omega t,$$

which gives $A = 0,\ B = \dfrac{F_0}{2\omega m}$, as required.

Chapter 4

Exercises 4.1

1. $\dfrac{1}{s^2}$.

3. $\dfrac{1}{s^2+1}$.

5. $\dfrac{1}{2(s-1)} - \dfrac{1}{2(s+1)}$.

7. $\dfrac{s-a}{(s-a)^2+b^2}$.

Exercises 4.2

1. $\dfrac{3}{s} - \dfrac{2}{s^2} + \dfrac{2}{s^3}$.

3. $\dfrac{3}{s} + \dfrac{4}{s-3} - \dfrac{2s}{s^2+4}$.

5. $\dfrac{10}{s^3} - \dfrac{4}{(s+3)^2+4}$.

7. $\dfrac{2s}{(s^2+1)^2} + \dfrac{2(s^2-4)}{(s^2+4)^2}$.

9. $\sinh \beta x = \dfrac{e^{\beta x} - e^{-\beta x}}{2}$.

11. $\dfrac{1}{2}\left[\dfrac{1}{s-3} + \dfrac{1}{s-2} - \dfrac{1}{s-1} + \dfrac{1}{s-4}\right]$.

15. $Y(s) = \dfrac{1}{s-2}$.

17. $Y(s) = \dfrac{2}{(s-2)(s+4)} - \dfrac{9}{(s^2+9)(s+4)} - \dfrac{3}{s+4}$.

19. $Y(s) = \dfrac{2}{(s+3)^2}$.

21. $Y(s) = \dfrac{3}{s(s-5)(s+3)} + \dfrac{4}{(s-5)(s+3)^2} + \dfrac{s-5}{(s-5)(s+3)}$.

23. Set $g(x) = \displaystyle\int_0^x f(t)\,dt$. Then $g'(x) = f(x)$ and $g(0) = 0$.

$$F(s) = \mathcal{L}[f(x)] = \mathcal{L}[g'(x)] = s\mathcal{L}[g(x)] - g(0) = s\mathcal{L}[g(x)].$$

Therefore, $\mathcal{L}[g(x)] = \dfrac{1}{s}F(s)$.

Exercises 4.3

1. $6e^{-7x}$.

3. $\frac{1}{5}\sin 5x$.

5. $e^{-4x}\cos x$.

7. $e^{-2x}\cos 2x + e^{-2x}\sin 2x.$

9. $2xe^{-2x} - e^{x}\cos x - e^{x}\sin x.$

11. $\frac{1}{2}e^{-x} - \frac{1}{2}\cos x + \frac{1}{2}\sin x.$

13. $\frac{1}{4} - \frac{1}{4}\cos 2x.$

15. $\frac{1}{2} - e^{x} + \frac{3}{2}e^{-2x}.$

17. $e^{2x} - 4e^{x} + 2x + 3.$

19. $\frac{2}{3}e^{-2x} + \frac{1}{3}e^{x}.$

21. $\frac{3}{2}e^{-x} - \frac{1}{2}\cos x + \frac{1}{2}\sin x.$

23. $e^{x}\sin x.$

25. $\frac{3}{4}e^{-x} + \frac{1}{4}e^{x} + \frac{1}{2}xe^{x}.$

27. $\frac{1}{4}e^{x} + xe^{-x} + x - 2.$

29. $e^{-2x} + e^{x}.$

31. $\alpha = \frac{1}{4}.$

33. $\beta = -\frac{26}{5}.$

35. $\frac{7}{4}e^{2(x-1)} - 3e^{x-1} + \frac{1}{2}x + \frac{3}{4}.$

Exercises 4.4

1. $\mathcal{L}[f(x)] = \dfrac{1}{s^2} - \dfrac{e^{-s}}{s} - \dfrac{e^{-2s}}{s} - \dfrac{e^{-2s}}{s^2}.$

3. $\mathcal{L}[f(x)] = \dfrac{2e^{-5s}}{s}.$

5. $\mathcal{L}[f(x)] = \dfrac{1}{s} + e^{-2s}\left(\dfrac{1}{s^2} - \dfrac{1}{s}\right) + e^{-4s}\left(\dfrac{1}{s+1} - \dfrac{2}{s} - \dfrac{1}{s^2}\right).$

7. $\dfrac{\dfrac{2}{s^2} - 2e^{-2s}\dfrac{1}{s} - 2e^{-2s}\dfrac{1}{s^2} - 2e^{-4s}\dfrac{1}{s}}{1 - e^{-4s}}.$

9. $\dfrac{2}{s^3} - e^{-3s}\dfrac{2}{s^3} - 3e^{-3s}\dfrac{1}{s^2}.$

11. $\dfrac{1}{s(s^2+1)}\left(e^{-\pi s} - e^{-2\pi s}\right).$

13. $\dfrac{1}{s^2}\left(1 - 2e^{-s} + 2e^{-3s} - e^{-4s}\right).$

15. $\sin x - \sin x\, u(x-\pi).$

17. $\cos x - \cos x\, u(x-\pi) + \sin x\, u(x-\pi)$.

19. $u(x-2) - e^{-(x-2)}u(x-2)$.

Exercises 4.5

1. $y = -\frac{1}{2} + \frac{5}{2}e^{2x} + u(x-1)\left[-\frac{1}{2} + \frac{1}{2}e^{2(x-1)}\right]$.

3. $y = 1 - \cos x + \sin x - u(x-1)[\cos(x-1) - 1]$.

5. $y = 1 - e^{-x} - xe^{-x} + u(x-2)\left[x - 4 + xe^{-(x-2)}\right]$.

7. $y = -\frac{1}{3} - \frac{1}{6}e^{3x} + \frac{1}{2}e^{x} + u(x-1)\left[\frac{1}{3} + \frac{1}{6}e^{3(x-1)} - \frac{1}{2}e^{x-1}\right]$.

9. $y = \frac{1}{4}e^{x} + \frac{11}{4}e^{-x} + \frac{3}{2}xe^{-x} + u(x-1)\left[xe^{-(x-1)} - 1\right]$.

Exercises 5.2

1. $x = 4, y = 1$.

3. $x = 4 - 2a,\ y = a,\ a$ any real number.

5. $x = -3,\ y = 1$.

7. No solution.

9. $x = 2a - 3,\ y = a,\ a$ any real number.

11. $x = \frac{3}{7}a + 1,\ y = \frac{5}{7}a - 1,\ z = a,\ a$ any real number.

13. No solution.

15. $x = 2,\ y = 1,\ z = 1$.

Exercises 5.3

1. matrix of coefficients: 3; augmented matrix: 3; $x = 5,\ y = 3,\ z = -1$.

3. matrix of coefficients: 2; augmented matrix: 2; $x = 4 - 2a,\ y = a,\ z = -2,\ a$ an real number.

5. matrix of coefficients: 3; augmented matrix: 3; $x_1 = -1,\ x_2 = -1 - 2a,\ x_3 = 3 + a,\ x_4 = a,\ a$ any real number.

7. matrix of coefficients: 3; augmented matrix: 3; $x_1 = 8 + 2a - 3b,\ x_2 = a,\ x_3 = 3 - 1 - 2b,\ x_4 = b,\ x_5 = -3,\ a,\ b$ any real numbers.

9. $x = 2,\ y = 5$.

11. $x = -3 - a,\ y = 2 + 2a,\ z = a,\ a$ any real number.

13. $x = \frac{10}{7},\ y = \frac{2}{7},\ z = \frac{3}{2}$.

15. $x_1 = 11 - 2a + b,\ x_2 = a,\ x_3 = 3 - b,\ x_4 = b,\ a,\ b$ any real numbers.

17. $x_1 = -2,\ x_2 = -5,\ x_3 = -1,\ x_4 = 5.$

19. $x_1 = 3 - 2a,\ x_2 = a,\ x_3 = 2,\ x_4 = 1.$

21. No solution.

23. $(i)\ k \neq -3,\ 2 \quad (ii)\ k = -3 \quad (iii)\ k = 2.$

25. (a) No (b) No (c) Yes

Exercises 5.4

1. Yes

3. No

5. No. The leading 1 in the last column is not the only nonzero in its column.

7. Yes

9. $x = 10,\ y = -9,\ z = -7$

11. $x = -3 - a,\ y = 2 + 2a,\ z = a,\ a$ any real number.

13. $x_1 = 11 - 2a + b,\ x_2 = a,\ x_3 = 3 - b,\ x_4 = b,\ a,\ b$ any real numbers.

15. $x_1 = 7 - 2a - b,\ x_2 = 1 + 3a - 4b,\ x_3 = a,\ x_4 = b,\ a,\ b$ any real numbers.

17. $x = y = 0.$

19. $x = y = z = 0.$

21. $x_1 = 2a - b,\ x_2 = -a + 4b,\ x_3 = a,\ x_4 = b,\ a,\ b$ any real numbers.

23. $x_1 = x_2 = x_3 = x_4 = 0.$

25. Consider the system

$$\begin{aligned} x + y &= 0 \\ 2x + 2y &= 0 \\ 3x + 3y &= 0 \end{aligned}$$

This system has the solutions $x = -a, y = a,\ a$ any real number.

27. $b = -3a,\ a$ any real number.

29. $a = 4.$

Exercises 5.5

1. (a) $\begin{pmatrix} 0 & 4 \\ 3 & 5 \\ 1 & -1 \end{pmatrix}$.

(c) $\begin{pmatrix} 8 & -4 \\ 1 & 4 \end{pmatrix}$.

(e) $\begin{pmatrix} 2 & 8 \\ 2 & 8 \\ 5 & -3 \end{pmatrix}$.

3. (a) $\begin{pmatrix} -4 & -3 \\ 28 & -6 \\ -20 & 24 \end{pmatrix}$.

(c) Not defined.

(e) $\begin{pmatrix} 1 & 3 \\ -3 & -12 \\ -41 & 21 \end{pmatrix}$.

5. (a) $c_{32} = 2$ (b) $c_{13} = 34$ (c) $d_{21} = 5$ (d) $d_{22} = 1$.

7. (a) $d_{22} = 6$ (b) $d_{12} = -4$ (c) $d_{23} = -18$.

11. (a) $AB = \begin{pmatrix} 4 & 7 & 10 \\ 0 & -5 & -14 \end{pmatrix}$, BA not defined.

(b) $AC = \begin{pmatrix} 14 & 5 \\ -2 & -3 \end{pmatrix}$, $CA = \begin{pmatrix} -1 & 14 \\ 5 & 12 \end{pmatrix}$.

(c) $AD = DA = \begin{pmatrix} 4 & 4 \\ -2 & 2 \end{pmatrix}$.

13. $A(BD) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -6 & 0 & 8 \\ 9 & 9 & -26 \end{pmatrix} = \begin{pmatrix} -6 & 0 & 8 \\ 9 & 9 & -26 \end{pmatrix}$

$(AB)D = \begin{pmatrix} 0 & -2 \\ 3 & 5 \end{pmatrix}\begin{pmatrix} -2 & 3 & -2 \\ 3 & 0 & -4 \end{pmatrix} = \begin{pmatrix} -6 & 0 & 8 \\ 9 & 9 & -26 \end{pmatrix}$.

15. (a) 3×3 (c) Does not exist (e) 2×3.

Exercises 5.6

1. $A^{-1} = \begin{pmatrix} 1/2 & 0 \\ -3/2 & 1 \end{pmatrix}$.

3. $A^{-1} = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$.

5. $A^{-1} = \begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$.

7. No inverse.

9. No inverse.

11. $\det A = \pm 1$.

13. $x = 5,\ y = 0$.

15. $x = \frac{9}{2},\ y = -5$.

17. $x = \frac{7}{9},\ y = \frac{1}{3},\ z = -\frac{5}{9}$.

19. -31.

21. -45.

23. 30.

25. -21.

27. -18.

29. 26.

31. $x = 0,\ 1,\ -3$.

33. $y = -\frac{25}{37}$.

35. Cramer's rule does not apply.

37. $x = 0$.

39. $\lambda = -4,\ 7$.

Exercises 5.7

3. Dependent; $(-4,\ 8,\ 9) = 2(1,\ -2,\ 3) + 3(-2,\ 4,\ 1)$.

5. Dependent; $(-2,\ 6,\ 3) = (1,\ -1,\ 3) + 2(0,\ 2,\ 3) - 3(1,\ -1,\ 2)$.

7. Dependent; $(7,\ -4,\ 1) = 3(1,\ -2,\ 1) + 2(2,\ 1,\ -1)$.

9. Dependent; $(4,\ -2,\ 0\ \ 2) = 2(2,\ -1,\ 0\ \ 1)$.

11. $b \neq -\frac{1}{3}$.

13. $b = 0,\ -7.$

17. No; a linearly dependent set can have linearly independent subsets. For example, $\{(1,\ -2,\ 3),\ (-2,\ 4,\ 1)\}$ is a linearly independent subset of $\{(1,\ -2,\ 3),\ (-2,\ 4,\ 1)\},\ (-4,\ 8,\ 9).$

19. $W(x) = -a$; linearly independent.

21. $W(x) = -2x^{-6}$; linearly independent.

23. $W(x) = e^{2x}(x-2)$; linearly independent.

25. (a) False (b) True (c) True.

Exercises 5.8

1. $2,\ \begin{pmatrix} 1 \\ 0 \end{pmatrix};\quad 3,\ \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

3. $-1,\ \begin{pmatrix} 1 \\ -1 \end{pmatrix};\quad 4,\ \begin{pmatrix} 2 \\ 3 \end{pmatrix}.$

5. $1,\ 1,\quad \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

7. $2,\ 2,\quad \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$

9. $2+i,\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} + i\begin{pmatrix} 0 \\ -1 \end{pmatrix};\quad 2-i,\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} - i\begin{pmatrix} 0 \\ -1 \end{pmatrix}.$

11. $8,\ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix};\quad 1,\ \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix};\quad 2,\ \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$

13. $1,\ 1,\ 1,\quad \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix},\ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$

15. $1+i,\ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + i\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix};\quad 1-i,\ \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - i\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix};\quad 0,\ \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$

17. $1,\ 1,\ 1,\quad \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$

19. $2+3i,\ \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix} + i\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix};\quad 2-3i,\ \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix} - i\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix};\quad 2,\ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$

21. $2, \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}; \quad 2, \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}; \quad 6, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}; \quad 4, \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}.$

Chapter 6

Exercises 6.1

1. $y = C_1 e^x + C_2 e^{2x} + C_3 e^{3x}$.

3. $y = C_1 e^{2x} + C_2 e^{-2x} + e^x[C_3 \cos 2x + C_4 \sin 2x]$.

5. $y = C_1 \cos x + C_2 \sin x + e^{2x}[C_3 \cos 3x + C_4 \sin 3x]$.

7. $y = C_1 + C_2 x + C_3 e^x + C_4 e^{-x} + C_5 \cos x + C_6 \sin x$.

9. $y = 2x$.

11. $y = \frac{1}{5} e^x - \frac{1}{5} \cos 3x - \frac{1}{15} \sin 3x$.

13. $y^{(4)} - 8y''' + 31y'' - 78y' + 90y = 0$.

15. $y^{(5)} - 2y^{(4)} - 2y''' - 2y'' - 3y' = 0$.

17. $y^{(5)} - 2y^{(4)} + y''' - 2y'' = 0$.

19. $y^{(4)} - y'' = 0$.

21. $y = C_1 e^{-x} + C_2 \cos x + C_3 \sin x + \frac{1}{4} e^x + 4$.

23. $y = C_1 \cos x + C_2 \sin x + C_3 x \cos x + C_4 x \sin x + 6 + \frac{1}{9} \cos 2x$.

25. $y = \frac{1}{72} e^{-x} \left[e^{3x}(2x - 1) + 3 \cos \sqrt{3}\, x + \sqrt{3} \sin \sqrt{3}\, x \right]$.

Exercises 6.2

1. $$\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -3 & t \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \sin 2t \end{pmatrix}.$$

3. $$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ e^t \end{pmatrix}.$$

5. $$\begin{aligned} x_1' &= 2x_1 - x_2 + e^{2t} \\ x_2' &= 3x_1 + 2e^{-t} \end{aligned} \; .$$

7. $$\begin{aligned} x_1' &= 2x_1 + 3x_2 - x_3 + e^t \\ x_2' &= -2x_1 + x_3 + 2e^{-t} \\ x_3' &= 2x_1 + 3x_2 + e^{2t} \end{aligned} \; .$$

9. $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \sin\, t \\ -2\cos\, t \end{pmatrix}.$

1. 1 $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -3 & 0 \\ 2 & -1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} 3e^{2t} \\ -2\cos\, t \\ t \end{pmatrix}.$

Exercises 6.3

1. Independent

3. Independent

5. Dependent

7. Dependent

9. Dependent

1. 1 (c) $\mathbf{x}(t) = c_1\mathbf{u} + c_2\mathbf{v}$, where $c_1,\ c_2$ are arbitrary constants.

(d) $\mathbf{x}(t) = -2\begin{pmatrix} e^{2t} \\ e^{2t} \end{pmatrix} + \begin{pmatrix} 3e^{3t} \\ 2e^{3t} \end{pmatrix}.$

1. 3 (b) $\mathbf{x}(t) = \begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 4te^{-t} \\ e^{-t} \\ 0 \end{pmatrix}.$

Exercises 6.4.1

1. $\mathbf{x}(t) = c_1e^{2t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_2e^{-3t}\begin{pmatrix} 4 \\ -1 \end{pmatrix}.$

3. $\mathbf{x}(t) = c_1\left[\cos\, 2t\begin{pmatrix} 1 \\ -1 \end{pmatrix} - \sin\, 2t\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] + c_2\left[\cos\, 2t\begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sin\, 2t\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right];$

$\mathbf{x}(t) = \cos\, 2t\begin{pmatrix} 1 \\ 3 \end{pmatrix} + \sin\, 2t\begin{pmatrix} 7 \\ -4 \end{pmatrix}.$

5. $\mathbf{x}(t) = c_1e^{-2t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + c_2e^{-2t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$

7. $\mathbf{x}(t) = c_1\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + c_2e^{-t}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + c_3e^{-3t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}.$

9. $\mathbf{x}(t) = c_1\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + c_2e^{-2t}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c_3e^{-2t}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}.$

11. $\mathbf{x}(t) = c_1 e^{2t}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + c_2 e^{2t}\left[\cos t\begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix} + \sin t\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}\right] + c_3 e^{2t}\left[\cos t\begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \sin t\begin{pmatrix} 5 \\ -2 \\ 5 \end{pmatrix}\right].$

Exercises 6.4.2

1. $\mathbf{v}_1 = e^{t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = e^{t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + te^{t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}.$

3. $\mathbf{v}_1 = e^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = e^{3t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + te^{3t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}.$

5. $\mathbf{v}_1 = e^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = e^{-t}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + te^{-t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}.$

7. $\mathbf{v}_1 = e^{4t}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = e^{-2t}\begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} + te^{-2t}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

9. $\mathbf{v}_1 = e^{-2t}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \quad \mathbf{v}_2 = e^{-2t}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + te^{-2t}\begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}.$

11. $\mathbf{v}_1 = e^{-t}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = e^{-t}\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + te^{-2t}\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}.$