Part I. Linear Algebra

Exercises 1.1

1. $x = 4,\ y = 1$

3. The set of points on the line $x + 2y = 4$

6. $x = 2,\ y = -1$

8. $x = -2,\ y = 3$

12. $x = -3 + 2y,\ z = 0,\ y$ any real number

15. $x = 2,\ y = 1,\ z = 1$

Exercises 1.2

3. Coefficient matrix: rank 2; augmented matrix: rank 2; $x = 4-2a,\ y = a,\ z = -2,\ a$ any real number

4. Coefficient matrix: rank 2; augmented matrix: rank 3; no solution

7. Coefficient matrix: rank 3; augmented matrix: rank 3; $x_1 = 8 + 2a - 3b,\ x_2 = a,\ x_3 = -1 - 2b,\ x_4 = b,\ x_5 = -3,\ a, b$ any real numbers

13. $x_1 = 10/7,\ x_2 = 2/7,\ x_3 = 3/2$

19. $x_1 = 3 - 2a,\ x_2 = a,\ x_3 = 2,\ x_4 = 1,\ a$ any real number

21. no solution

23. $(i)\ k \neq -3, 2$ $\quad (ii)\ k = -3$ $\quad (iii)\ k = 2$

25. (a) no (b) no (c) yes

26. $x_1 = -71,\ y_1 = 19,\ z_1 = 8;\quad x_2 = 29,\ y_2 = -7,\ z_2 = -3;\quad x_3 = 12,\ y_3 = -4,\ z_3 = -1$

Exercises 1.3

3. yes

4. yes

5. No; the leading 1 in the 3,5-position is not the only nonzero entry in its column.

8. yes

11. $x = -3 - a,\ y = 2 + 2a,\ z = a,\ a$ any real number

13. $x_1 = 11 - 2a + b,\ x_2 = a,\ x_3 = 3 - b,\ x_4 = b,\ a, b$ any real numbers

18. $x = y = z = 0,$ the trivial solution

21. $x_1 = 2a - b,\ x_2 = -a + 4b,\ x_3 = a,\ x_4 = b,\ a, b$ any real numbers

28. $a = b\ \ b$ any real number

29. $a = 4$

Exercises 1.4

3. (a) $\begin{pmatrix} -4 & -3 \\ 28 & -6 \\ -20 & 24 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 3 \\ -1 & -12 \\ -41 & 21 \end{pmatrix}$

4. (a) 3×2 (b) 4×2 (c) does not exist (d) 3×2 (e) 4×2
(f) 3×2 (g) does not exist

6. (b) -12 (c) 4

9. For $i = 1, 2, \ldots, n,$ the element in the 2^{nd} row, i^{th} column is

$$c_{2i} = a_{21}b_{1i} + a_{22}b_{2i} + \cdots + a_{2n}b_{ni} = 0$$

since $a_{21} = a_{22} = \cdots = a_{2n} = 0.$

10. For $j = 1, 2, \ldots, n,$ the element in the j^{th} row, 3^{rd} column is

$$c_{j3} = a_{j1}b_{13} + a_{j2}b_{23} + \cdots + a_{jn}b_{n3} = 0$$

since $b_{13} = b_{23} = \cdots = b_{n3} = 0.$

14. (d) 6×6 (e) does not exist

Exercises 1.5

3. $\begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$

5. $\begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$

7. No inverse

12. $x = 5,\ y = 0$

15. $x = 17,\ y = 5,\ z = -7$

Exercises 1.6

2. 0

5. 30

8. -10

11. 26

13. $x = 0,\ x = 1,\ x = -3$

15. $y = -25/37$

17. Cramer's rule does not apply; the determinant of the matrix of coefficients is 0.

18. $y = 1/4$

21. $\lambda = -4,\ 7.$ For $\lambda = -4,\ x = -\frac{6}{5}\,a, y = a,\ a$ any real number; for $\lambda = 7,\ x = y = a,\ a$ any real number

22. $\lambda = 1,\ 1 + \sqrt{5},\ 1 - \sqrt{5}.$ For $\lambda = 1,\ x = 0,\ y = -\frac{1}{2}\,a,\ z = a,\ a$ any real number

Part II. Probability

Exercises 2.1.1

2. (a) $n(A \cap B) = 20$ (b) $n(A \cap B^c) = 5,$ (c) $n(A^c \cap B^c) = 40$ (d) $n[(A \cup B)^c] = 40$

3.

$\cap$	A	A^c	Tot
B	20	60	80
B^c	40	30	70
Tot	60	90	150

5. 450

7. (a) 80 (b) 18 (c) 28 (d) 34

8. (a) 760 (b) 1290 (c) 720

9. (a) $n(A \cap B \cap C^c) = 5$ (b) $n(A \cup B) = 27$
(c) $n[(A \cup B \cup C)^c] = 7$ (d) $n[(A \cup B)^c \cap C] = 10$

11. (a) 190 (b) 90 (c) 210

12. 100

Exercises 2.1.2

2. 84

3. (a) 10^5 (b) $10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30,240$ (c) $9 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 27,216$

4. (a) $2 \cdot (26)^3 = 35,152$ (b) $2 \cdot 25 \cdot 19 \cdot 18 = 10,260$ (c) $2 \cdot 5 \cdot 3 \cdot 24 \cdot 23 = 16,560$

5. (a) $18 \cdot 17 \cdot 16 \cdot 15 = 73,440$ (b) $10 \cdot 9 \cdot 8 \cdot 7 = 5040$

6. (a) $5 \cdot 5 \cdot 4 \cdot 3 = 300$ (b) $5 \cdot 6 \cdot 6 \cdot 3 = 540$ (c) $5 \cdot 6 \cdot 9 = 270$ (d) $5 \cdot 5 \cdot 5 \cdot 5 = 625$

8. (a) 48 (b) 60 (c) 72

9. There are only $26^2 = 676$ distinct pairs of initials.

10. (a) 72 (b) 450 (c) 176

12. (a) 9,360,000 (b) 468,000 (c) 8,564,400

13. (a) 8 (b) 6 shoes, 6 slacks, 12 shirts

Exercises 2.2

1. (a) $P_{15,3} = 15 \cdot 14 \cdot 13 = 2,730$ (b) $C_{15,5} = 3003$

3. (a) $C_{7,2} = 21$ (b) $2 \cdot C_{7,3} = 70$ (c) $C_{9,4} = 126$

5. (a) $C_{52,5} = 2,598,960$ (b) $C_{13,5} = 1,287$ (c) $C_{12,5} = 792$

(d) $C_{13,3} \cdot C_{13,2} = 22,308$

6. (a) 5! (b) 4!.

8. (a) $C_{13,5} = 1287$ (b) $C_{8,3} \cdot C_{5,2} = 560$ (c) $C_{8,3} \cdot C_{5,2} + C_{8,4} \cdot C_{5,1} + C_{8,5} = 966$

9. (a) 4 (b) $4 \cdot 9 = 36$ (c) $13 \cdot 48 = 624$ (d) $13 \cdot C_{4,3} \cdot 12 \cdot C_{4,2} = 3,744$

10. (a) $C_{15,3} \cdot 20 = 9,100$ (b) $C_{15,2} \cdot C_{20,2} = 19,950$ (c) $C_{35,4} = 52,360$

(d) $C_{15,3} \cdot 20 + C_{15,4} = 10,465$

Exercises 2.3

3. (a) $(2a - b)^5 = 32a^5 - 80a^4b + 80a^3b^2 - 40a^2b^3 + 10ab^4 - b^5$

(b) $(x - 1)^6 = x^6 - 6x^5 + 15x^4 - 20x^3 - 15x^2 + 6x - 1$

4. (c) $90x^3$

5. (a) $C_{11,6}\, a^5b^6 = 462a^5b^6$

Exercises 3.1

3. (b) $S = \{BBB,\ BBG,\ BG,\ GB,\ GGB,\ GGG\}$ (c) $E = \{GGB,\ GGG\}$

4. (a) $S = \{(1,1,5,10), (1,1,10), (1,5,1,10), (1,5,10), (1,10), (5,1,1,10), (5,1,10), (5,10), (10)\}$

(b) $E = \{(1,1,5,10), (1,5,1,10), (5,1,1,10)\}$ (c) $F = \{(1,1,10), (1,10), (10)\}$

6. (a) 24 (b) 18 (c) 8 (d) 12

7. (a) $S = \{(1,1), (1,2), \cdots, (1,6), (2,1) \cdots, (3,6)\};\ n(S) = 18$

(b) $n(E) = 9$ (c) $n(F) = 3$

8. (a) $C_{52,5} = 2,598,960$ (b) 48 (c) $4 \cdot C_{13,5} = 5,148$

(d) $13 \cdot 48 = 624$ (e) $13 \cdot C_{4,3} \cdot 12 \cdot C_{4,2} = 3,744$

Exercises 3.2

3. (a) no; cannot have a negative probability.

(b) no; the sum of the probabilities is not 1.

(c) acceptable

4. $P(1) = P(3) = P(5) = \frac{1}{9}$; $P(2) = P(4) = P(6) = \frac{2}{9}$.

5. (a) $P(BBG) = \frac{1}{8}$ (b) $E = \{BBG, BGB, GBB\}$; $P(E) = \frac{3}{8}$

(c) $F = \{BBG, BGB, BGG, GBB, GBG, GGB\}$, $P(F) = \frac{6}{8} = \frac{3}{4}$

7. (a) $P(12) = \frac{1}{36}$ (b) $P(5) = \frac{4}{36} = \frac{1}{9}$ (c) $P(> 8) = \frac{10}{36} = \frac{5}{18}$

(d) $P(< 7) = \frac{15}{36} = \frac{5}{12}$ (e) $P(odd) = \frac{18}{36} = \frac{1}{2}$ (f) $P(\text{prime}) = \frac{15}{36} = \frac{5}{12}$

(g) $P(\text{divisible by } 3) = \frac{12}{36} = \frac{1}{3}$

8. (a) $P(6) = \frac{1}{9}$ (b) $P(3) = \frac{2}{9}$ (c) $P(\text{odd number}) = \frac{4}{9}$ (d) $P(7) = 0$

(e) $P(> 3) = \frac{6}{9} = \frac{2}{3}$

10. (a) $P(\text{odd}) = \frac{18}{38} = \frac{9}{19}$ (b) $P(> 24) = \frac{12}{38} = \frac{6}{19}$ (c) $P(< 15) = \frac{16}{38} = \frac{8}{19}$

(d) $P(\text{prime}) = \frac{11}{38}$

11. (a) $\dfrac{C_{13,5}}{C_{52,5}} \cong 0.0004952$ (b) $\dfrac{C_{26,5}}{C_{52,5}} \cong 0.02531$ (c) $\dfrac{13 \cdot 48}{C_{52,5}} \cong 0.0002401$

(d) $\dfrac{C_{4,3} \cdot C_{4,2}}{C_{52,5}} \cong 0.000009234$ (e) $\dfrac{13 \cdot C_{4,3} \cdot 12 \cdot C_{4,2}}{C_{52,5}} = 0.0014406$

Exercises 3.3

3. $E =$ probability that at least two people have same birth month.

$$P(E) = 1 - \frac{12 \cdot 11 \cdot 10}{12^3} \cong 0.236$$

4. 14%

5. $P(B^c \cup A^c) = P([a \cap B]^c) = 0.9;$ $P(A \cap B^c) = 0.3$

7. 1:4

Exercises 3.4

1. $P(H_2|H_1) = \dfrac{12}{51} = \dfrac{4}{17}$ The events are dependent since $P(H_2) = \dfrac{1}{4}$

3. $P(Sp|J) = \dfrac{1}{4}$; $P(Sp) = \dfrac{1}{4}$. The events are independent. ǎ 6

4. $P(R_2|H_1) = \dfrac{25}{51} = \dfrac{4}{17}$ The events are dependent since $P(R_2) = \dfrac{1}{2}$

5. (a) $P(A|B) = \dfrac{1}{2}$; $P(A) = \dfrac{1}{4}$; the events are dependent.

(b) $P(A|B) = \dfrac{1}{2}$; $P(A) = \dfrac{1}{2}$; the events are independent.

6. (a) $P(R_2) = \dfrac{14}{49}$ (b) $P(R_2) = \dfrac{12}{42} = \dfrac{2}{7}$ (c) $P(\text{same color}) = \dfrac{29}{49}$

8. (a) $P(16) = \dfrac{1}{6}$ (b) $P(17) = \dfrac{1}{4}$ (c) $P(2^{nd}\text{draw}) = \dfrac{1}{4}$

9. $D =$ the patient has the defect, $E =$ the test is positive; $P(D|E) \cong 0.19$

11. (a) $P(Tb|+) = 0.807$ (b) $P(nTb|+) = 0.193$ (c) $P(Tb|-) = 0.0035$

12. $P(\text{male}) = \dfrac{205}{457}$, $P(\text{very sat.}|\text{male}) = \dfrac{75}{205}$, $P(\text{barely sat.}|\text{female}) = \dfrac{87}{176}$

14. (a) $P(U_1)|W) = \dfrac{5}{19}$ (b) $P(U_2)|R) = \dfrac{3}{8}$

15. (a) $P(RU_1|RU_2) = \dfrac{35}{47}$ (b) $P(RU_1|WU_2) = \dfrac{35}{53}$

16. $P(A|B) = P(A) \Rightarrow \dfrac{P(A \cap B}{P(B)} = P(A) \Rightarrow P(A \cap B) = P(A)P(B)$ So A and B are independent.

17. $P(A|B) + P(A^c|B) = \dfrac{P(A \cap B}{P(B)} + \dfrac{P(A^c \cap B}{P(B)} = \dfrac{P(A \cap B) + P(A^c \cap B)}{P(B)} = \dfrac{P(B)}{P(B)} = 1$

18. If $B \subset A$, then $A \cap B = B$.

Exercises 3.5

1.

x	1	2	3	4	5	6
$p[X = x]$	1/36	3/36	5/36	7/36	9/36	11/36

2. $E(X) = 4.472\cdots$

5. \$25

6. \$1456

8. In pounds sterling, $E(X) = -0.325$, which is half the value in dollars. If Y is the value in dollars, then $X = \frac{1}{2}Y$ and $E(X) = E(\frac{1}{2}Y) = \frac{1}{2}E(Y)$.

9. $E(X) = \$999.35 = \$1000 - 0.65$ If Y is the original random variable and $X = 1000 + Y$, then $E(X) = E(1000 + Y) = 1000 + E(Y)$.

10. $E(X) = \dfrac{91}{6}$. If Y is the number of spots, then $E(Y) = \dfrac{7}{2}$. Now, $E(X) = E(Y^2) = \frac{91}{6} \neq [E(Y)]^2$.

Part III. Differential Equations

Chapter 1

Exercises 1.1

3. y_1 is a solution, y_2 is not.

4. Both y and z are solutions.

7. $r = -4,\ r = 2$

8. $r = 3$

10. $r = 1,\ r = 2$

11. $r = 2$

14. (b) $y = x^2 \ln x$ (c) and (d) No solutions; $\ln 0$ is not defined.

15. (a) $y = C_1 \sin x$ for any constant C_1. (b) The initial conditions imply $C_1 = C_2 = 0$.

17. $xy' - 2y + 6 = 0$

19. $y'' + y' - 2y = 0$

21. $x^2y'' - 2xy' + 2y = 0$

Chapter 2

Exercises 2.1

2. $y = x + Cx^2$

3. $y = \dfrac{\sin x}{x^2} + \dfrac{C}{x^2}$

5. $y = x(\ln x)^2 + Cx$

7. $y = x^2e^x - ex^2$

9. $y^{1/2} = Ce^{2x} - e^x$ or $y = (Ce^{2x} - e^x)^2$

Exercises 2.2

2. $y = -\ln\,(e^x - xe^x + C)$

3. $\dfrac{y-1}{y+1} = Ce^{x^2-2x}$

6. $y = \ln\left[\ln\left(\dfrac{e}{1+e}[1+e^x]\right)\right]$

7. $y + \ln\, y = \frac{1}{3}x^3 - x - 5$

9. $y = \dfrac{1}{Cx + 1 + \ln\, x}$

10. $y = Cxe^{y/x} - xe^{y/x}\,\ln\, x - x$

Exercises 2.3

1. $x^2 + 3y^2 = C$

2. $\dfrac{(x-1)^2}{2} + (y-2)^2 = C$

4. (a) $A(t) = 50e^{\frac{t}{2}\,\ln(9/10)} = 50\left(\dfrac{9}{10}\right)^{t/2}$ (b) 40.5 (c) 13.16 hours

6. (a) $P(t) = 4.5e^{0.02310t}$ (b) 48.19 years (c) 6.17 billion (approx.)

7. 132 years; 2112

8. 8:52 pm

Chapter 3

Exercises 3.2

2. Yes, their Wronskian is non-zero.

3. (a) $r = -1,\ 4$ (b) $\{e^{-x},\ e^{4x}\}$; $y = C_1e^{-x} + C_2e^{4x}$ (c) $y = \frac{4}{5}e^{-x} + \frac{1}{5}e^{4x}$

4. (a) $r = -1,\ 4$ (b) $\{x^{-1},\ x^4\}$; $y = C_1x^{-1} + C_2x^4$ (c) $y = \frac{9}{5}x^{-1} + \frac{1}{5}x^4$

(d) $y \equiv 0$

5. (a) $y = x + 1$ and $y = x^2 + 1$ are solutions and they are independent.

(b) $y = C_1(x+1) + C_2(x^2+1)$

7. $\{x,\ x^2\}$

9. $\alpha\delta - \beta\gamma \neq 0$

10. Set $u = \dfrac{y_2}{y_1}$ and calculate u'. $W \equiv 0 \Rightarrow u' = 0 \Rightarrow u = C$ constant.

Exercises 3.3

1. $y = C_1 e^{6x} + C_2 e^{7x}$

2. $y = C_1 e^{5x} + C_2 x e^{5x}$

5. $y = C_1 e^{-x} \cos x\sqrt{2} + C_2 e^{-x} \sin x\sqrt{2}$

7. $y = -e^{x} \cos x$

9. $y'' - 6y' + 9y = 0$

10. $y'' + 2y' + 10y = 0$

11. (a) $y = \dfrac{\alpha+2}{3} e^{2x} + \dfrac{2\alpha - 2}{3} e^{-x};\ \ \alpha = -2$

(b) $y = \dfrac{\beta+2}{3} e^{2x} + \dfrac{4-\beta}{3} e^{-x};\ \beta = -2$

12. (a) $ga < 0$ (b) $\alpha > 1$

15. $y = C_1 x^4 + C_2 x^{-2}$

16. $y = C_1\, x^2 + C_2\, x^2 \ln x$