Assignment 10 Chapters 7 & 8.

Due date: **July 2**

Section 7.1. 1, 3

Section 7.2. 1, 2

Section 7.4. 2, 3, 6

Section 8.1. 1, 2, 3, 4, 5, 6, 9, 10

Problems to be turned in for grading.

1. (a) Find the coordinates of the vector $v = (1, 4)$ in the orthonormal basis $\mathcal{V}$

$$v_1 = \frac{1}{\sqrt{5}}(1, 2) \quad \text{and} \quad v_2 = \frac{1}{\sqrt{5}}(2, -1).$$

   (b) Let $A = \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}$. Find $[A]_{\mathcal{V}}$.

2. Use Gram-Schmidt orthonormalization to find an orthonormal basis for the subspace of $\mathbb{R}^3$ spanned by the vectors

$$w_1 = (1, 1, 1) \quad \text{and} \quad w_2 = (0, 1, 1).$$

   Extend this basis to an orthonormal basis of $\mathbb{R}^3$.

3. (Computer exercise) Use Gram-Schmidt orthonormalization to find an orthonormal basis for the subspace of $\mathbb{R}^5$ spanned by the vectors

$$w1 = (2, 1, 3, 5, 7) \quad w2 = (2, -1, 5, 2, 3) \quad \text{and} \quad w3 = (10, 1, -23, 2, 3).$$

   Extend this basis to an orthonormal basis of $\mathbb{R}^5$.

4. Let

$$A = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

   be the general real 2×2 symmetric matrix.

   (a) Prove directly using the discriminant of the characteristic polynomial that A has real eigenvalues.

   (b) Show that A has equal eigenvalues only if A is a scalar multiple of I_2.

In Exercises 5 and 6, determine whether the given matrix A is diagonalizable. If it is, find a matrix P such that $P^{-1}AP$ is diagonal.

5. $A = \begin{pmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{pmatrix}$.

6. $A = \begin{pmatrix} -1 & 4 & 2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{pmatrix}$.

7. According to Theorem 7.4.1 an $n \times n$ symmetric matrix has only real eigenvalues and the corresponding eigenvectors can be chosen so as to form an orthonormal basis for $\mathbb{R}^n$. Let

$$A = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}.$$

   Find the eigenvalues of A and find an orthonormal set of eigenvectors (which forms a basis for $\mathbb{R}^3$).

8. Let A be an $n \times n$ matrix with a real eigenvalue λ and associated eigenvector v. Assume that all other eigenvalues of A are different from λ. Let B be an $n \times n$ matrix that commutes with A; that is, $AB = BA$. Show that v is also an eigenvector for B.

9. Assuming that $b \neq 0$, find a matrix that orthogonally diagonalizes

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix}.$$