

Assignment 2. Chapter 2.

Due date: June 5

Section 2.1. 2, 4, 5, 6, 9 – 11

Section 2.2. 1, 3, 4, 5, 9

Section 2.3. 1 – 6, 8 – 14, 20, 21

Section 2.4. 1– 7, 10

Problems to be turned in for grading.

1. (Computer exercise) Suppose that the four substances S_1, S_2, S_3, S_4 contain the following percentages of vitamins A, B, C and F by weight

Vitamin	S_1	S_2	S_3	S_4
A	25%	19%	20%	3%
B	2%	14%	2%	14%
C	8%	4%	1%	0%
F	25%	31%	25%	16%

Mix the substances S_1, S_2, S_3 and S_4 so that the resulting mixture contains precisely 3.85 grams of vitamin A, 2.30 grams of vitamin B, 0.80 grams of vitamin C, and 5.95 grams of vitamin F. How many grams of each substance have to be contained in the mixture?

Discuss what happens if we require that the resulting mixture contains 2.00 grams of vitamin B instead of 2.30 grams.

2. Determine graphically the geometry of the set of solutions to the system of equations in the three unknowns x, y, z :

$$\begin{aligned}x + 3z &= 1 \\3x - z &= 1 \\z &= 2\end{aligned}$$

by sketching the plane of solutions for each equation individually. Describe in words why there are no solutions to this system. (Use MatLab graphics to verify your sketch. Note that you should enter the last equation as $\mathbf{z} = 2 - 0*\mathbf{x} - 0*\mathbf{y}$ and the first two equations with $0*\mathbf{y}$ terms. Try different views — but include `view([0 1 0])` as one view.)

3. (Computer exercise) Find a cubic polynomial

$$p(x) = ax^3 + bx^2 + cx + d$$

so that $p(1) = 2, p(2) = 3, p'(-1) = -1,$ and $p'(3) = 1.$

4. Given the system of linear equations
- $$\begin{aligned}2x - 3y &= 6 \\4x - 6y &= 7\end{aligned}$$

- (a) Show that the system does not have a solution.
(b) Write the coefficient matrix A for the system.
(c) What can you say about the two row vectors of A ? What can you say about the two column vectors of A ?

5. (a) Use Gaussian elimination to solve the system

$$\begin{aligned}x + 2y - 3z &= 1 \\2x + 5y - 8z &= 4 \\3x + 8y - 13z &= 7\end{aligned}$$

- (b) The coefficient matrix of the given system is $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 5 & -8 \\ 3 & 8 & -13 \end{pmatrix}$ Show that $(3, 8, -13)$ can be written in the form

$$(3, 8, -13) = r(1, 2, -3) + s(2, 5, -8)$$

- (c) Can you write each row vector in terms of the other two?

6. Determine the values of k so that the system of equations

$$\begin{aligned}x + y - z &= 1 \\2x + 3y + kz &= 3 \\x + ky + 3z &= 2\end{aligned}$$

has: (a) a unique solution, (b) no solution, (c) infinitely many solutions.

7. (Computer Exercise) Given the systems of equations

- (a)

$$\begin{aligned}x + 2y - z &= 3 \\2x + 5y - 4z &= 5 \\3x + 4y + 2z &= 12\end{aligned}$$

- (b)

$$\begin{aligned}3x - 2y + z &= -7 \\2x + y - 4z &= 0 \\x + y - 3z &= 1\end{aligned}$$

- (c)

$$\begin{aligned}x - 2y + 3z &= 2 \\2x - 3y + z &= 1 \\3x - 4y - z &= 1\end{aligned}$$

Write the augmented matrix for each system, find the reduced echelon form of each augmented matrix and, in each case, give the rank of the coefficient matrix and the rank of the augmented matrix. In general, what can conclude about the rank of the coefficient matrix versus the rank of the augmented matrix and the existence of solutions of a system of linear equations?