

Assignment 3. Chapter 3.

Due date: June 8

Section 3.1. 2, 4, 5, 8, 11, 12, 13

Section 3.2. 3, 4, 5, 6, 8, 9, 18, 22

Section 3.3. 3, 4, 5, 6, 8, 9,

Section 3.4. 2, 3

Problems to be turned in for grading.

1. (Computer exercise) Find all solutions to

$$\begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 1 & 5 & 7 \\ 3 & 4 & 4 & 11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 14 \\ 17 \\ 31 \end{pmatrix}.$$

2. (Computer exercise) Let A be a 3×3 matrix. Find A so that

$$\begin{aligned} A \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} &= \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \\ A \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} &= \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix} \\ A \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} &= \begin{pmatrix} 5 \\ 1 \\ 1 \end{pmatrix}. \end{aligned}$$

Hint: Rewrite these three conditions as a system of linear equations in the nine entries of A .

3. What 2×2 matrix rotates the plane clockwise $\frac{2\pi}{3}$ radians?
4. The matrix $\begin{pmatrix} 4 & -3 \\ 3 & 4 \end{pmatrix}$ rotates the plane counterclockwise through an angle θ and then dilates the plane by a factor of c . Find θ and c . **Hint:** See Section 3.2, Exercise 13.
5. Let $L : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear map. Prove that $L(\mathbf{0}) = \mathbf{0}$.

In problems 6 – 8, determine whether or not each transformation is linear.

6. $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1 + 1, 2x_2, x_1 + x_2)$.
7. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2, x_1 + x_2 + x_3)$.
8. $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2, x_3^2)$.

9. Given the system of equations

$$x_1 - 3x_2 - 2x_3 + 4x_4 = 5$$

$$3x_1 - 8x_2 - 3x_3 + 8x_4 = 18$$

$$2x_1 - 3x_2 + 5x_3 - 4x_4 = 19$$

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- (a) Write the system in the matrix form $A\mathbf{x} = \mathbf{b}$.
- (b) Find the set of all solutions of the homogeneous system $A\mathbf{x} = \mathbf{0}$.
- (c) Find a single solution of the inhomogeneous system $A\mathbf{x} = \mathbf{b}$.
- (d) Use your answers to (b) and (c) to find all solutions of the given system.