MATH 3321(H)

Homework Assignment #3

Due date: Friday, February 10, 2012

NAME (Print): _____

PeopleSoft ID: _____

Instructions:

- Print out this file and complete all the problems.
- Write your solutions in the space provided.
- Use a blue or black pen, or a pencil. Xerox copies of your homework are not acceptable.
- Your homework must be complete, neatly written and stapled.
- Submit your completed homework in class on the due date.

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1. Let $y = y(x)$ be the solution of the initial-value problem

$$y' + 2\,y = 3, \quad y(0) = 1.$$

Find $\lim_{x\to\infty} y(x)$.

2. Let $y = y(x)$ be the solution of the initial-value problem

$$xy' + 2\,y = \frac{2\ln\,x}{x}, \quad y(1) = 1.$$

Find $\lim\limits_{x\to\infty} y(x)$.

3. Find the solution $y = y(x)$ of the initial-value problem.

$y' + ay = b, \quad y(0) = 0, \quad$ where a and b are positive constants

and then calculate $\lim\limits_{x\to\infty} y(x)$.

4. Consider the first order linear differential equation

$$y' + p(x)y = q(x). \tag{L}$$

Show that if $u = u(x)$ is a solution of the reduced equation

$$y' + p(x)y = 0$$

and $z = z(x)$ is a solution of (L), then $y = Cu(x) + z(x)$ is the general solution of (L).

5. Given the differential equation

$$y' + 2y = \cos x. \tag{a}$$

(a) Set $z(x) = A\cos x + B\sin x$. Find values for A and B so that z a solution of (a).

(b) Find a solution of the reduced equation $y' + 2y = 0$.

(c) Use the result in Problem 4 to find the general solution of (a).

6. Suppose that q is continuous on (a, ∞) and k is a constant.

(a) Derive a formula for the solution of the initial-value problem

$$y' + ky = q(x), \quad y(c) = \alpha \tag{b}$$

where $c \in (a, \infty)$.

(b) Suppose that $k > 0$ and $\lim_{x \to \infty} q(x) = M$. Show that if $y = y(x)$ is the solution of the initial-value problem (b), then $\lim_{x \to \infty} = \frac{M}{\alpha}$.

7. A metal ball at room temperature $20^o\,C$ is dropped into a container of boiling water $(100^o\,C)$. Given that the temperature of the ball increases 2^o in 2 seconds, find:

   (a) The temperature of the ball after 6 seconds in the boiling water.

   (b) How long it will take for the temperature of the ball to reach $90^o\,C$.

8. A disease is infecting a herd of 100 cows living on Bud Smith's ranch. Let $P(t)$ be the number of sick cows t days after the outbreak. Suppose that 15 cows had the disease initially, and suppose that the disease is spreading at a rate proportional to the number of cows who do not have the disease.

   (a) Give the mathematical model (initial-value problem) for P.

   (b) Find the general solution of the differential equation in (a).

   (c) Find the particular solution that satisfies the initial condition.

9. A 1000-gallon tank, initially full of water, develops a leak at the bottom. Let $A(t)$ be the amount of water in the tank at time t. Suppose that 500 gallons of water leak out in the first 30 minutes, and suppose that the water drains off a rate proportional to the product of the time elapsed and the square root of the amount of water present.

   (a) Give the mathematical model (initial-value problem) for A.

   (b) Find the general solution of the differential equation in (a).

   (c) Find the particular solution that satisfies the initial condition.

10. Bacteria in a colony are born at a rate proportional to the number present and die at a rate proportional to the square of the number present. Therefore, if $P(t)$ is the number of bacteria present at time t, then

$$\frac{dP}{dt} = aP - bP^2,$$

where a is the birth rate and b is the death rate.

(a) Find the number of bacteria in the colony at time t if $P(0) = M$.

(b) What is the limiting size of the population, i.e., what is $\lim_{t\to\infty} P(t)$?