Assignment 4. Chapter 3.

Due date: June 11

Section 3.5. 2, 3, 4, 6, 8, 9, 13, 14

Section 3.6. 4, 8

Section 3.7. 2, 5, 6, 10, 11

Section 3.8. 1, 2, 3, 6, 8, 12, 13, 14

Problems to be turned in for grading.

1. Let
$$A = \begin{pmatrix} 2 & 5 \\ 1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} a & 3 \\ b & 2 \end{pmatrix}.$$
For which values of a and b does $AB = BA$?

2. Recall that a square matrix C is *upper triangular* if $c_{ij} = 0$ when $i > j$. Show that the matrix product of two upper triangular $n \times n$ matrices is also upper triangular.

3. Let A, B, C be general $n \times n$ matrices. Simplify the expression $A^{-1}(BA^{-1})^{-1}(CB^{-1})^{-1}$.

4. Let A be an $n \times n$ matrix that satisfies
$$A^m + a_{m-1}A^{m-1} + \cdots + a_1 A + I_n = 0.$$
Show that A is invertible.

5. For which values of a, b, c is the matrix
$$A = \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix}$$
invertible? Find A^{-1} when it exists.

6. (Computer exercise) Try to compute the inverse of the matrix
$$C = \begin{pmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 0 & 2 & 1 \end{pmatrix}. \qquad (3.7.7^*)$$
in MatLab using the command `inv`. What happens — can you explain the outcome?

   Now compute the inverse of the matrix
$$\begin{pmatrix} 1 & \epsilon & 3 \\ -1 & 2 & -2 \\ 0 & 2 & 1 \end{pmatrix}$$
for some nonzero numbers ϵ of your choice. What can be observed in the inverse if ϵ is very small? What happens when ϵ tends to zero?

7. Let A be a 2×2 matrix having integer entries. Find a condition on the entries of A that guarantees that A^{-1} has integer entries.

8. If a and b are real numbers and $ab = 0$, then either $a = 0$ or $b = 0$, or both numbers are 0. Show by example that this property does not hold for matrices. That is, find two non-zero matrices A and B such that $AB = 0$.

9. Prove that if A is an invertible matrix such that

   (a) $AB = AC$, then $B = C$.

   (b) $AB = 0$, then $B = 0$.

10. What is incorrect about the following "proof?"

    Let $A\mathbf{x} = \mathbf{b}$ be a system of linear equations, and let $\mathbf{x}_1$ and $\mathbf{x}_2$ be solutions. Then

    $$A\mathbf{x}_1 = \mathbf{b} \qquad \text{and} \qquad A\mathbf{x}_2 = \mathbf{b}.$$

    Therefore

    $$\begin{aligned} A\mathbf{x}_1 &= A\mathbf{x}_2 \\ \mathbf{x}_1 &= \mathbf{x}_2. \end{aligned}$$