**Assignment 5.** Chapter 4.

**Due date: June 15**

Section 4.1. 1, 2, 4, 6, 8, 14

Section 4.2. 3, 5, 6, 7, 17, 20, 21, 26, 27, 28, 32, 33

**Problems to be turned in for grading.**

1. **True or False**. If the statement is true, give reason(s); it is false, give a counterexample.
   (a) The determinant of $I + A$ is $1 + \det A$.
   (b) The determinant of $ABC$ is $\det A \det B \det C$.
   (c) The determinant of $4A$ is $4 \det A$.
2. Two $n \times n$ matrices $A$ and $B$ are similar if there exists an $n \times n$ matrix $P$ such that $B = P^{-1}AP$. Show that the determinants of similar $n \times n$ matrices are equal.
3. Let
$$A = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & 0 \\ 1 & 5 & 3 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$
   (a) For what values of $\lambda$ is $\det(\lambda A - B) = 0$?
   (b) Is there a vector $x$ for which $Ax = Bx$?
4. Suppose that two $n \times p$ matrices $A$ and $B$ are row equivalent. Show that there is an invertible $n \times n$ matrix $P$ such that $B = PA$.
5. Solve the system of equations by finding the inverse of the matrix of coefficients.
$$\begin{aligned} x - 3y &= 2 \\ y + z &= -2 \\ 2x - y + 4z &= 1 \end{aligned}$$
6. Find the values of $x$ such that $\begin{vmatrix} x & 0 & 2 \\ 2x & x-1 & 4 \\ -x & x-1 & x+1 \end{vmatrix} = 0.$
7. Determine the values of $\lambda$ for which the system
$$\begin{aligned} (\lambda + 4)x + 4y + 2z &= 0 \\ 4x + (5 - \lambda)y + 2z &= 0 \\ 2x + 2y + (2 - \lambda)z &= 0 \end{aligned}$$
   has nontrivial solutions. Find the solutions for each value of $\lambda$.
8. Prove that if $A$ is invertible, then $\det(A^{-1}) = \dfrac{1}{\det A}$.
9. Let $A$ and $B$ be $n \times n$ matrices. Prove that if $AB$ is invertible, then each of $A$ and $B$ is invertible. Is the converse true? That is, if $A$ and $B$ are each invertible, does it follow that $AB$ is invertible