Assignment 7 Chapter 5.

Due date: June 22

Section 5.1. 3, 5, 6, 7, 8, 9, 10, 11, 13

Section 5.2. 2, 5, 7, 9, 11, 12, 13, 14, 15

Section 5.3. 1, 2, 5

Problems to be turned in for grading.

1. Let V be a vector space and let W_1 and W_2 be subspaces. Show that the intersection $W_1 \cap W_2$ is also a subspace of V.
2. Let $S = \{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$. Is S a subspace of $\mathbb{R}^3$? Justify your answer.
3. Let $M_n = \{A |\, A \text{ is an } n \times n \text{ matrix}\}$. Show that M_n is a vector space under the operations of matrix addition and scalar multiplication.
4. Let $S_n = \{A \in M_n |\, A \text{ is symmetric}\}$. Show that S is a subspace of M_n.
5. Let $D = \{A \in M_2 |\, \det A = 0\}$. Is D a subspace of M_2? Justify your answer.
6. The matrix
$$B = \begin{pmatrix} 1 & -1 & 0 & 5 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{pmatrix}$$
is the coefficient matrix of a homogeneous system of linear equations. Write the solution set as a span of vectors.
7. Let $W \subset \mathbb{R}^4$ be the subspace that is spanned by the vectors
$$w_1 = (-1, 2, 1, 5) \quad \text{and} \quad w_2 = (2, 1, 3, 0).$$
Find a linear system of two equations such that $W = \text{span}\{w_1, w_2\}$ is the set of solutions of this system.
8. Let V be a vector space and let $v, w \in V$ be vectors. Show that
$$\text{span}\{v, w\} = \text{span}\{v, w, v + 3w\}.$$
9. (Computer exercise) let $W \subset \mathbb{R}^5$ be the subspace spanned by the vectors
$$w_1 = (2, 0, -1, 3, 4), \quad w_2 = (1, 0, 0, -1, 2), \quad w_3 = (0, 1, 0, 0, -1).$$
Use MatLab to decide whether the given vectors are elements of W.
   (a) $v_1 = (2, 1, -2, 8, 3)$.
   (b) $v_2 = (-1, 12, 3, -14, -1)$.
10. (a) Determine whether the vectors $v_1 = (1, 1)$ and $v_2 = (-2, 1)$ span $\mathbb{R}^2$.
    (b) Determine whether the vectors $v_1 = (2, 1, -1)$, $v_2 = (3, -1, 1)$, $v_3 = (1, 4, -4)$ span $\mathbb{R}^3$.