Assignment 8 Chapter 5.

Due date: **June 25**

Section 5.4. 1 – 6, 8, 10

Section 5.4A. 1, 2, 3, 6, 7, 9, 14, 15

Section 5.5. 1 – 5

Section 5.6. 1, 2, 3, 9, 10

Problems to be turned in for grading.

1. Show that any set of vectors in $\mathbb{R}^n$ that contains the zero vector is a linearly dependent set.

2. For which values of b are the vectors $(2,\ -b)$, $(2b+6,\ 4b)$ linearly dependent?

3. Let $S = \{\mathbf{v}_1,\ \mathbf{v}_2,\ \ldots,\ \mathbf{v}_k\}$ be a linearly independent set of vectors. Prove that every non-empty subset of S is also linearly independent. Suppose that $T = \{\mathbf{v}_1,\ \mathbf{v}_2,\ \ldots,\ \mathbf{v}_m\}$ is a linearly dependent set. Is every non-empty subset of T linearly dependent? Prove or give a counter-example.

4. (Computer exercise) Perform the following experiments.

   (a) Use MatLab to choose randomly three column vectors in $\mathbb{R}^3$. The MatLab commands to choose these vectors are:

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   y1 = rand(3,1)
   y2 = rand(3,1)
   y3 = rand(3,1)
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   Use the methods of this section to determine whether these vectors are linearly independent or linearly dependent.

   (b) Now perform this exercise five times and record the number of times a linearly independent set of vectors is chosen and the number of times a linearly dependent set is chosen.

   (c) What do you conclude about choosing three vectors at random in $\mathbb{R}^3$ in terms of linear independence versus linear dependence?

   (d) Repeat the experiment in (b) — but this time randomly choose four vectors in $\mathbb{R}^3$ to be in your set. What do conclude about choosing four vectors at random in $\mathbb{R}^3$ in terms of linear independence versus linear dependence?

5. Show that the functions $f_0(x) = 1,\ f_1(x) = x,\ f_2(x) = x^2, \ldots,\ f_k(x) = x^k$ are linearly independent.

6. Show that the set of vectors of the form $(a, 2b, a+3b)$ form a subspace of $\mathbb{R}^3$. What is the dimension of the subspace?

7. Let V be a vector space of dimension greater than 1. State with a brief explanation (give an example, cite a theorem or definition, give a proof, etc.) whether the following statements are true or false.

(a) A basis for V can include the zero vector.

(b) There is more than one basis for V.

(c) There exists a set which spans V but is not linearly independent.

(d) There exists a set that is linearly independent but does not span V.

(e) If there are three linearly independent vectors in V, then the dimension of V must be greater than V.

8. Let

$$A = \begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 1 & 5 & 7 \\ 3 & 4 & 4 & 11 \end{pmatrix}.$$

(a) Find a basis for the subspace $\mathcal{C} \subset \mathbb{R}^3$ spanned by the columns of A.

(b) Find a basis for the subspace $\mathcal{R} \subset \mathbb{R}^4$ spanned by the rows of A.

(c) What is the relationship between $\dim \mathcal{C}$ and $\dim \mathcal{R}$?