Assignment 9 Chapter 6.

Due date: **June 29**

Section 6.1. 1, 7

Section 6.2. 1, 2, 3

Section 6.3. 1, 3, 5, 7

Section 6.4. 1

Problems to be turned in for grading.

1. (Computer exercise) Let
$$A = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 0 & -1 & 3 & 1 \\ 2 & -1 & 1 & 0 \\ -1 & 0 & 7 & 4 \end{pmatrix}.$$
   (a) Compute rank(A) and exhibit a basis for the row space of A.
   (b) Find a basis for the column space of A.
   (c) Find all solutions to the homogeneous equation $Ax = 0$.
   (d) Does
   $$Ax = \begin{pmatrix} 4 \\ 2 \\ 2 \\ 1 \end{pmatrix}$$
   have a solution?

2. Let $v_1 = (1, 1)$ and $v_2 = (-1, 0)$.
   (a) Verify that $\{v_1, v_2\}$ is a basis for $\mathbb{R}^2$.
   (b) Construct a matrix A of the linear mapping $L : \mathbb{R}^2 \to \mathbb{R}^2$ defined by
   $$L(v_1) = (0, 2) \quad \text{and} \quad L(v_2) = (-1, -1)$$

3. Let $v_1 = (1, 3)$ and $v_2 = (-2, 4)$.
   (a) Verify that $\{v_1, v_2\}$ is a basis for $\mathbb{R}^2$.
   (b) Construct a matrix A of the linear mapping $L : \mathbb{R}^2 \to \mathbb{R}^3$ defined by
   $$L(v_1) = (7, -1, 0) \quad L(v_2) = (6, 2, 0)$$

4. Let $w_1 = (1, 1, 1)$, $w_2 = (2, 2, 0)$, $w_3 = (3, 0, 0)$.
   (a) Verify that $\{w_1, w_2, w_3\}$ is a basis for $\mathbb{R}^3$.
   (b) Construct a matrix A of the linear mapping $L : \mathbb{R}^3 \to \mathbb{R}^3$ defined by
   $$L(w_1) = (0, 0, 2) \quad L(w_2) = (2, -1, -1) \quad L(w_3) = (6, 3, 1)$$

5. Let $w_1 = (1, 2)$ and $w_2 = (0, 1)$ be a basis for $\mathbb{R}^2$. Let $L_A : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map given by the matrix

$$A = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$$

in standard coordinates. Find the matrix $[L]_{\mathcal{W}}$.

6. (Computer exercise) Let A be the 4×4 matrix

$$A = \begin{pmatrix} 2 & 1 & 4 & 6 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 4 \\ 2 & 1 & 1 & 5 \end{pmatrix}$$

and let $\mathcal{W} = \{w_1, w_2, w_3, w_4\}$ where

$$\begin{aligned} w_1 &= (1, 2, 3, 4) \\ w_2 &= (0, -1, 1, 3) \\ w_3 &= (2, 0, 0, 1) \\ w_4 &= (-1, 1, 3, 0) \end{aligned}$$

Verify that $\mathcal{W}$ is a basis of $\mathbb{R}^4$ and compute the matrix associated to A in the $\mathcal{W}$ basis.

7. Consider the bases $W = \{w_1,\ w_2\}$ and $Z = \{z_1,\ z_2\}$ for $\mathbb{R}^2$ where

$$w_1 = \begin{pmatrix} 2 \\ 2 \end{pmatrix},\ w_2 = \begin{pmatrix} 4 \\ -1 \end{pmatrix};\quad z_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix},\ z_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}.$$

(a) Find the transition matrix C_{WZ}.

(b) Find the transition matrix C_{ZW}.

(c) Calculate $C_{WZ}C_{ZW}$. What do you conclude?