

Problem 1.

- (a) (i) $\overrightarrow{AB} = (2, 4, 1)$, $\overrightarrow{AC} = (-3, 3, -6)$, $\overrightarrow{BC} = (-5, -1, -7)$
 $\overrightarrow{AB} \cdot \overrightarrow{AC} = 0 \implies \overrightarrow{AB} \perp \overrightarrow{AC}$, A, B, C are the vertices of a right triangle.
- (ii) hypotenuse: $\|\overrightarrow{BC}\| = \sqrt{75} = 5\sqrt{3}$
- (iii) $(x - \frac{1}{2})^2 + (y - \frac{3}{2})^2 + (z + \frac{3}{2})^2 = 0$
- (b) (i) $3(2\mathbf{a} - \mathbf{b} + \mathbf{c}) = 3\mathbf{i} - 9\mathbf{k}$
- (ii) $\text{proj}_{\mathbf{b}} \mathbf{c} = \frac{\mathbf{b} \cdot \mathbf{c}}{\|\mathbf{b}\|^2} \mathbf{b} = \frac{9}{14}(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = \frac{9}{7}\mathbf{i} + \frac{27}{14}\mathbf{j} - \frac{9}{14}\mathbf{k}$
- (iii) $\mathbf{b} \times \mathbf{a} = -3\mathbf{i} - 6\mathbf{k}$; unit vector: $\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{k}$
- (iv) $V = 15$

Problem 2.

- (a) normal vector for plane: $\mathbf{N} = (3, -1, 4)$; direction vector for line: $\mathbf{d} = (-2, 2, 2)$
 $\mathbf{N} \cdot \mathbf{d} = 0 \implies \mathbf{N} \perp \mathbf{d}$; P and L are parallel.
- (b) $\frac{-3-1}{-2} = \frac{0+4}{2} \neq \frac{5+3}{2}$; A does not lie on L ; $d(A, L) = \frac{4\sqrt{6}}{3}$
- (c) $x = -3 + 3t$, $y = -t$, $z = 5 + 4t$
- (d) $P(-51/13, 4/13, 49/13)$

Problem 3.

- (a) $(x+2) + 4(y-1) - z = 0$
- (b) $3(-2) - 1(1) + 2(0) = -7 \neq 4$; A is not on P ; $d(A, P_2) = \frac{11}{\sqrt{14}}$
- (c) $\mathbf{i} + 4\mathbf{j} - \mathbf{k} \neq \lambda(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$; P_1 and P_2 are not parallel.
direction vector for line of intersection: $\mathbf{N}_1 \times \mathbf{N}_2 = 7\mathbf{i} - 5\mathbf{j} - 13\mathbf{k}$;
point on line of intersection: $P(0, 24/7, 26/7)$
symmetric equations for line of intersection: $\frac{x}{7} = \frac{y - 24/7}{-5} = \frac{z - 26/7}{-13}$
- (d) $\cos \theta = \frac{|\mathbf{N}_1 \times \mathbf{N}_2|}{\|\mathbf{N}_1 \times \mathbf{N}_2\|} = \frac{3}{\sqrt{252}} \cong 0.1890$; $\theta \cong 1.3 \text{ rad.} \cong 79.11^\circ$

Problem 4.

- (a) (i) $\lim_{t \rightarrow 3} \mathbf{f}(t) = 7\mathbf{i} + \mathbf{j}$
(ii) $[\mathbf{f}(t) \cdot \mathbf{g}(t)]' = 6t^2 + 2t + 3\pi \sin \pi t$
- (b) (i) $\mathbf{v}(t) = \mathbf{r}'(t) = (-e^{-t} \sin t + e^{-t} \cos t)\mathbf{i} - (e^{-t} \cos t + e^{-t} \sin t)\mathbf{j} + 3\mathbf{k}$
(ii) speed: $\|\mathbf{v}(t)\| = \sqrt{2e^{-2t} + 9}$
(iii) $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t) = -2e^{-t} \cos t \mathbf{i} + 2e^{-t} \sin t \mathbf{j}$
(iv) $\lim_{t \rightarrow \infty} \mathbf{v}(t) = 3\mathbf{k}$

Problem 5.

- (a) (i) $L(C) = \int_0^3 \sqrt{t^2 + t^4} dt = \int_0^3 t\sqrt{1+t^2} dt = \frac{1}{3} \left[(1+t^2)^{3/2} \right]_0^3 = \frac{1}{3} [(10)^{3/2} - 1]$
(ii) $\kappa = \frac{\sqrt{2}}{4}$
- (b) (i) $x = 6 + 2t, y = 9 + 6t, z = 9 + 9t$
(ii) $L(C) = \int_0^3 \sqrt{4 + 4t^2 + t^4} dt = \int_0^4 (2 + t^2) dt = 15$
(iii) $\mathbf{T}(1) = \frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}; \quad \mathbf{N}(1) = -\frac{2}{3}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$
(iv) normal vector: $\mathbf{T}(1) \times \mathbf{N}(1) = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}; \quad \text{point: } \mathbf{r}(1) = 2\mathbf{i} + \mathbf{j} + \frac{1}{3}\mathbf{k}, \text{ i.e. } (2, 1, \frac{1}{3});$
osculating plane: $(x - 2) - 2(y - 1) + 2(z - \frac{1}{3}) = 0$

Problem 6.

- (a) $\mathbf{T}(t) = \frac{1}{2} \sin t \mathbf{i} + \frac{1}{2} \cos t \mathbf{j} + \frac{1}{2} \sqrt{3} \mathbf{k}; \quad \mathbf{N}(t) = -\sin t \mathbf{i} + \cos t \mathbf{j}$
- (c) $\kappa = \frac{1}{4t}$
- (d) $\mathbf{a}(t) = 2\mathbf{T} + t\mathbf{N}; \quad a_{\mathbf{T}} = 2, \quad a_{\mathbf{n}} = t$