

**Problem 0.** (a) Find the domain and range of the following functions:

$$(i) f(x, y) = \ln \sqrt{1 + x^2 + y^2} \quad (ii) F(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 1}}$$

(b) Identify the level curves/surfaces of each of the following functions:

$$(i) f(x, y) = e^{4x^2 - 3y^2} \quad (ii) F(x, y, z) = 2x + 3y + 6$$

(c) A closed box of length  $x$ , width  $y$ , and height  $z$ , is to have a total surface area of 20 sq. ft. Express the volume of the box as a function of  $x$  and  $z$ .

**Problem 1.** (a) Set  $f(x, y) = \frac{2xy + 1}{\sqrt{9 - x^2 - y^2}}$ .

(i) What is the domain of  $f$ ?

(ii) Does  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exist? If so, what is the limit?

(b) Let  $f(x, y) = \frac{2x^2y}{x^4 + y^2}$

(i) Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  if  $(x, y) \rightarrow (0, 0)$  along the  $x$ -axis.

(ii) Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  if  $(x, y) \rightarrow (0, 0)$  along the  $y$ -axis.

(iii) Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  if  $(x, y) \rightarrow (0, 0)$  along the line  $y = mx$ .

(iv) Find  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  if  $(x, y) \rightarrow (0, 0)$  along the parabola  $y = x^2$ .

(v) Does  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$  exist?

**Problem 2.**

(a) Let  $f(x, y) = y^2 e^{xy} + \frac{x}{y}$ . Calculate  $f_{xx}$  and  $f_{yx}$ .

(b) Let  $z = \ln \sqrt{x^2 + y^2}$ . Show that  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 1$

(c) Let  $u = x^2 - 2y^2 + z^3$  where  $x = \sin t$ ,  $y = e^{2t}$ ,  $z = 3t$ . Calculate  $\frac{du}{dt}$ .

(d) Let  $z = e^{2x} \ln y$  where  $x = u^2 - 2v$  and  $y = v^2 - 2u$ . Calculate  $\frac{\partial z}{\partial u}$  and  $\frac{\partial z}{\partial v}$

**Problem 3.** Let  $f(x, y) = x^2 y e^{x-1} + 2xy^2$  and  $F(x, y, z) = x^2 + 3yz + 4xy$ .

(a) (i) Find the gradient of  $f$ .

(ii) Determine the direction in which  $f$  decreases most rapidly at the point  $(1, -1)$ . At what rate

is  $f$  decreasing?

- (b) (i) Find the gradient of  $F$ .  
(ii) Find the directional derivative of  $F$  at the point  $(1, 1, -5)$  in the direction of the vector  $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \sqrt{3}\mathbf{k}$ .
- (c) Find an equation for the tangent plane to the level surface  $F(x, y, z) = 3$  at the point  $(3, -1, -2)$ .
- (d) Find an equation for the tangent plane and scalar parametric equations for the normal line to the surface  $z = f(x, y)$  at the point  $(1, -1, 1)$ .

**Problem 4.** In each of the following, determine whether  $\mathbf{F}$  is the gradient of a function  $f$ . If it is, find  $f$ .

- (a)  $\mathbf{F}(x, y) = (3x^2y^2 + 3y + x)\mathbf{i} + (2x^3y + 3xy - \sqrt{y})\mathbf{j}$ .  
(b)  $\mathbf{F}(x, y) = (2x e^y + e^{2x} - 3)\mathbf{i} + (x^2 e^y + \cos 2y + 1)\mathbf{j}$ .

**Problem 5.**

- (a) Find the stationary points of  $f(x, y) = \frac{1}{3}x^3 + \frac{1}{2}y^2 + 2xy - 7x - y + 3$ .  
(b) For each stationary point  $P$  found in (a), determine whether  $f$  has a local maximum, a local minimum, or a saddle point at  $P$ .

**Problem 6.**

- (a) Find the absolute maximum and absolute minimum values of  $f(x, y) = x^2 + y^2 - 2x - 2y + 2$  on the closed disk  $D : x^2 + y^2 \leq 4$ .  
(b) Find the absolute maximum and absolute minimum values of  $f(x, y) = 2 + 2x + 2y - x^2 - y^2$  on the closed triangular region bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x + y = 9$ .

**Problem 7.**

- (a) According to U.S. Postal Service regulations, the length plus the girth (perimeter of a cross-section) of a package cannot exceed 108 inches. What are the dimensions of the rectangular box of maximum volume that is acceptable for mailing? What is the maximum volume?  
(b) A rectangular box without a top is to have a volume of 12 cubic feet. The materials used to construct

the box cost \$3 per square foot for the sides and \$4 per square foot for the bottom. What dimensions will yield the minimum cost?