

Problem 1. (a) Set $f(x, y) = \frac{2xy + 1}{\sqrt{9 - x^2 - y^2}}$.

(i) What is the domain of f ?

(ii) Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist? If so, what is the limit?

(b) Let $f(x, y) = \frac{2x^2y}{x^4 + y^2}$

(i) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ if $(x, y) \rightarrow (0, 0)$ along the x -axis.

(ii) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ if $(x, y) \rightarrow (0, 0)$ along the y -axis.

(iii) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ if $(x, y) \rightarrow (0, 0)$ along the line $y = mx$.

(iv) Find $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ if $(x, y) \rightarrow (0, 0)$ along the parabola $y = x^2$.

(v) Does $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ exist ?

Problem 2.

(a) Let $f(x, y) = y^2 e^{xy} + \frac{x}{y}$. Calculate f_{xx} and f_{yx} .

(b) Let $z = \ln \sqrt{x^2 + y^2}$. Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 1$

(c) Let $u = x^2 - 2y^2 + z^3$ where $x = \sin t$, $y = e^{2t}$, $z = 3t$. Calculate $\frac{du}{dt}$.

(d) Let $z = e^{2x} \ln y$ where $x = u^2 - 2v$ and $y = v^2 - 2u$. Calculate $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$

Problem 3. Let $f(x, y) = x^2 y e^{x-1} + 2xy^2$ and $F(x, y, z) = x^2 + 3yz + 4xy$.

(a) (i) Find the gradient of f .

(ii) Determine the direction in which f decreases most rapidly at the point $(1, -1)$. At what rate is f decreasing?

(b) (i) Find the gradient of F .

(ii) Find the directional derivative of F at the point $(1, 1, -5)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - \sqrt{3}\mathbf{k}$.

(c) Find an equation for the tangent plane to the level surface $F(x, y, z) = 3$ at the point $(3, -1, -2)$.

(d) Find an equation for the tangent plane and scalar parametric equations for the normal line to the

surface $z = f(x, y)$ at the point $(1, -1, 1)$.

Problem 4. In each of the following, determine whether \mathbf{F} is the gradient of a function f . If it is, find f .

(a) $\mathbf{F}(x, y) = (3x^2y^2 + 3y + x) \mathbf{i} + (2x^3y + 3xy - \sqrt{y}) \mathbf{j}$.

(b) $\mathbf{F}(x, y) = (2x e^y + e^{2x} - 3) \mathbf{i} + (x^2 e^y + \cos 2y + 1) \mathbf{j}$.

Problem 5.

(a) Find the stationary points of $f(x, y) = \frac{1}{3}x^3 + \frac{1}{2}y^2 + 2xy - 7x - y + 3$.

(b) For each stationary point P found in (a), determine whether f has a local maximum, a local minimum, or a saddle point at P .

Problem 6.

(a) Find the absolute maximum and absolute minimum values of $f(x, y) = x^2 + y^2 - 2x - 2y + 2$ on the closed disk $D : x^2 + y^2 \leq 4$.

(b) Find the absolute maximum and absolute minimum values of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ on the closed triangular region bounded by the lines $x = 0$, $y = 0$, $x + y = 9$.

Problem 7.

(a) According to U.S. Postal Service regulations, the length plus the girth (perimeter of a cross-section) of a package cannot exceed 108 inches. What are the dimensions of the rectangular box of maximum volume that is acceptable for mailing? What is the maximum volume?

(b) A rectangular box without a top is to have a volume of 12 cubic feet. The materials used to construct the box cost \$3 per square foot for the sides and \$4 per square foot for the bottom. What dimensions will yield the minimum cost?