1. The general solution of $y'' - 3y' - 10y = 0$ is:

   (a) $y = C_1 e^{-5x} + C_2 e^{-2x}$
   (b) $y = C_1 e^{-5x} + C_2 e^{2x}$
   (c) $y = C_1 e^{5x} + C_2 e^{-2x}$
   (d) $y = C_1 e^{5x} + C_2 e^{2x}$
   (e) None of the above.

2. The solution of the initial-value problem $y'' - 8y' + 16y = 0,\ y(0) = 1,\ y'(0) = 2$ is:

   (a) $y = e^{-4x} - 6xe^{-4x}$
   (b) $y = \frac{1}{2}e^{-4x} + \frac{3}{2}e^{4x}$
   (c) $y = e^{4x} - 2xe^{4x}$
   (d) $y = e^{-4x} + 6xe^{-4x}$
   (e) None of the above.

3. A fundamental set of solutions of $y'' - 4y' + 13y = 0$ is:

   (a) $\{e^{2x}\cos 3x,\ e^{2x}\sin 3x\}$
   (b) $\{e^{3x}\cos 2x,\ e^{3x}\sin 2x\}$
   (c) $\{e^{-2x}\cos 3x,\ e^{-2x}\sin 3x\}$
   (d) $\{e^{-3x}\cos 2x,\ e^{-3x}\sin 2x\}$
   (e) None of the above.

4. $y'' - \dfrac{1}{x}y' - \dfrac{8}{x^2}y = 0$ has solutions of the form $y = x^r$. A fundamental set of solutions of the equation is:

   (a) $\{x^8,\ x^{-1}\}$
   (b) $\{x^{-4},\ x^2\}$
   (c) $\{x^4,\ x^{-2}\}$
   (d) $\{x^8,\ x\}$
   (e) None of the above.

5. $y = C_1 e^x + C_2 e^{-5x}$ is the general solution of a second order linear differential equation. The equation is:

(a) $y'' - 4y' - 5y = 0$
(b) $y'' - 4y' - 4y = 0$
(c) $y'' + 4y' - 5y = 0$
(d) $y'' + 4y' + 5y = 0$
(e) None of the above.

6. $y = xe^{-3x}$ is a solution of a second order linear differential equation with constant coefficients. The equation is:

(a) $y'' - 6y' + 9y = 0$
(b) $y'' + 3y' = 0$
(c) $y'' + 6y' + 9y = 0$
(d) $y'' - 6y' - 9y = 0$
(e) None of the above.

7. $y = e^{-3x} \cos x$ is a solution of a second order linear differential equation with constant coefficients. The equation is:

(a) $y'' + 6y' + 10y = 0$
(b) $y'' - 6y' + 10y = 0$
(c) $y'' - 2y' + 10y = 0$
(d) $y'' + 2y' - 10y = 0$
(e) None of the above.

8. If $y = y(x)$ is the solution of the initial-value problem $y'' + 2y' + 5y = 0,\ y(0) = y'(0) = 1$, then $\lim_{x \to \infty} y(x) =$

(a) does not exist
(b) ∞
(c) 1
(d) 0
(e) None of the above.

9. Let $y = y(x)$ be the solution of the initial-value problem:

$$y'' - y' - 6y = 0, \quad y(0) = \alpha,\ y'(0) = 4.$$

A value of α such that $y = y(x)$ satisfies $\lim_{x\to\infty} y(x) = 0$ is:

(a) $\alpha = 1$
(b) $\alpha = -1$
(c) $\alpha = 0$
(d) $\alpha = -2$
(e) None of the above.

10. The general solution of $xy' - 2y = x^4e^{2x}$ is

(a) $y = 2x^3e^{2x} - \frac{1}{2}x^2e^{2x} + Cx^2$
(b) $y = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C/x^2$
(c) $y = \frac{1}{2}x^3e^{2x} - \frac{1}{4}x^2e^{2x} + Cx^2$
(d) $y = x^3e^{2x} - \frac{1}{2}x^2e^{2x} + C/x$
(e) None of the above.

11. The family of orthogonal trajectories of $y^3 = Cx^2 + 2$ is:

(a) $3(x-1)^2 + 2y^2 + \dfrac{12}{y} = C$
(b) $2y^2 - 3(x-1)^2 - 6\ln y = C$
(c) $3(x-1)^2 + 2y^2 + 12\ln y = C$
(d) $3(x-1)^2 + 2y^2 - \dfrac{12}{y} = C$
(e) None of the above.

12. A 1000-gallon tank, initially full of water, develops a leak at the bottom. Let $A(t)$, be the amount of water in the tank t minutes after the leak develops. Suppose the water drains off a rate proportional to the product of the time elapsed and the amount of that has leaked out. The mathematical model for this problem is:

(a) $\dfrac{dA}{dt} = k(1000 - A),\ A(0) = 1000$
(b) $\dfrac{dA}{dt} = ktA,\ A(0) = 1000$
(c) $\dfrac{dA}{dt} = kt(1000 - A),\ A(0) = 1000$
(d) $\dfrac{dA}{dt} = kt(A + 1000),\ A(0) = 1000$
(e) None of the above.