1. A group of engineers design a vibrating mechanical system that falls into the undamped free vibration category. They then decide to apply a periodic external force. What is the probability that resonance will occur?

   (a) 1
   (b) π
   (c) 0
   (d) ∞
   (e) None of the above.

2. An object in simple harmonic motion has period $\dfrac{\pi}{2}$. At time $t = 0$, $y(0) = 0$, $y'(0) = -2$. The equation of motion is:

   (a) $y = 2\sin\left(2t + \frac{1}{2}\pi\right)$
   (b) $y = \frac{1}{2}\sin(4t + \pi)$
   (c) $y = 2\sin(4t + \pi)$
   (d) $y = \sin(2t + \pi)$
   (e) None of the above.

3. The transient solution of the vibrating system

$$y'' + 2y' + 2y = \cos 2t, \quad y(0) = 0,\ y'(0) = 2$$

   is:

   (a) $y = C_1 e^{-t}\cos t + C_2 e^{-t}\sin t - \frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$
   (b) $y = -\frac{1}{10}e^{-t}\cos t + \frac{1}{2}e^{-t}\sin t$
   (c) $y = -\frac{1}{10}\cos 2t + \frac{1}{5}\sin 2t$
   (d) $y = \frac{1}{10}e^{-t}\cos t + \frac{17}{10}e^{-t}\sin t$
   (e) None of the above.

4. The steady-state solution of the vibrating system

$$y'' + 4y' + 8y = 6 + 10e^{2t}, \quad y(0) = 2,\ y'(0) = 1$$

   is:

   (a) $z = \frac{3}{4}C_1 e^{-2t}\cos 2t + C_2 e^{-2t}\sin 2t - \frac{3}{4}e^{-t}\sin 2t$
   (b) $z = \frac{3}{4}C_1 e^{-2t}\cos 2t - \frac{3}{4}e^{-t}\sin 2t + \frac{4}{3} + \frac{1}{2}e^{2t}$
   (c) $z = \frac{3}{4} + \frac{1}{2}e^{2t}$
   (d) $z = \frac{4}{3} + \frac{1}{2}e^{2t}$
   (e) None of the above.

5. The general solution of $y^{(4)} + 21y'' - 100y = 0$ is:

   (a) $y = C_1 \cos 5x + C_2 \sin 5x + C_3 e^{2x} + C_4 e^{-2x}$
   (b) $y = C_1 \cos 5x + C_2 \sin 5x + C_3 \cos 2x + C_4 \sin 2x$
   (c) $y = C_1 \cos 2x + C_2 \sin 2x + C_3 e^{5x} + C_4 x e^{5x}$
   (d) $y = C_1 \cos 2x + C_2 \sin 2x + C_3 e^{5x} + C_4 e^{-5x}$
   (e) None of the above.

6. The general solution of $y^{(4)} - y''' - 3y'' + 17y' - 30y = 0$ is:

   (a) $y = C_1 \cos 3x + C_2 \sin 3x + C_3 e^{-3x} + C_4 e^{2x}$
   (b) $y = C_1 e^x \cos 2x + C_2 e^x \sin 2x + C_3 e^{-3x} + C_4 e^{2x}$
   (c) $y = C_1 \cos 2x + C_2 \sin 2x + C_3 e^{-3x} + C_4 e^{2x}$
   (d) $y = C_1 e^x \cos 2x + C_2 e^x \sin 2x + C_3 e^{3x} + C_4 e^{-2x}$
   (e) None of the above.

7. The general solution of $y''' - y'' - 8y' + 12y = 0$ is:

   (a) $y = C_1 e^{-2x} + C_2 x e^{-2x} + C_3 e^{-3x}$
   (b) $y = C_1 e^{2x} + C_2 x e^{2x} + C_3 e^{3x}$
   (c) $y = C_1 e^{3x} + C_2 e^{-2x} + C_3 x e^{-2x}$
   (d) $y = C_1 e^{-3x} + C_2 e^{2x} + C_3 x e^{2x}$
   (e) None of the above.

8. The order of the linear, constant coefficient, homogeneous equation of least order that has

$$y = 3e^{2x} - 7e^{3x} + e^{-2x} \cos 4x - 4x$$

   as a solution is:

   (a) 7
   (b) 6
   (c) 5
   (d) 4
   (e) None of the above.

9. The linear, constant coefficient, homogeneous equation of least order that has

$$y = 3\cos 3x + 4\sin 3x + 2xe^{-x}$$

as a solution is:

(a) $y^{(4)} + 2y''' + 10y'' + 18y' + 9y = 0$
(b) $y^{(4)} - 2y''' + 10y'' - 18y' + 9y = 0$
(c) $y^{(5)} + 2y''' + 10y'' - 8y' + 9y = 0$
(d) $y^{(4)} + 2y''' + 10y'' + 10y' + 9y = 0$
(e) None of the above.

10. The linear, constant coefficient, homogeneous equation of least order that has

$$y = 5e^{-x}\cos 2x - 3e^{x} + 2x$$

as a solution is:

(a) $y^{(4)} - y''' + 3y'' - 5y' = 0$
(b) $y^{(5)} - 2y''' + 10y'' - 5y' = 0$
(c) $y^{(5)} + y^{(4)} + 3y''' - 5y'' = 0$
(d) $y^{(4)} + y''' + 3y'' - 5y' = 0$
(e) None of the above.

11. A linear, constant coefficient, homogeneous equation that has

$$y = 2\cos 2x + 5e^{-2x} - 3e^{2x}$$

as a solution is:

(a) $y^{(5)} + 16y' = 0$
(b) $y^{(4)} + 16y = 0$
(c) $y^{(5)} - 16y' = 0$
(d) $y''' - 16y = 0$
(e) None of the above.

12. A particular solution of $y''' - 5y'' + 8y' - 4y = 6e^{2x} + 4e^{x}$ is:

(a) $z = 3xe^{2x} - 4xe^{x}$
(b) $z = -\frac{3}{2}x^2e^{2x} + 4xe^{x}$
(c) $z = \frac{3}{2}x^2e^{2x} - 2xe^{x}$
(d) $z = 3x^2e^{2x} + 4xe^{x}$
(e) None of the above.

13. The general solution of

$$y^{(4)} + 4y''' + 13y'' + 36y' + 36y = -5e^{-2x} + 7\sin\, 2x + 4$$

will have the form:

(a) $y = C_1e^{-2x} + C_2xe^{-2x} + C_3\cos\, 3x + C_4\sin\, 3x + Ax^2e^{-2x} + B\cos 2x + C\sin 2x + D$
(b) $y = C_1e^{2x} + C_2xe^{2x} + C_3\cos\, 3x + C_4\sin\, 3x + Ae^{2x} + B\cos\, 2x + C\sin\, 2x + D$
(c) $y = C_1e^{-2x} + C_2xe^{-2x} + C_3\cos\, 3x + C_4\sin\, 3x + Axe^{2x} + B\cos 2x + C\sin 2x + Dx$
(d) $y = C_1e^{2x} + C_2xe^{2x} + C_3\cos\, 3x + C_4\sin\, 3x + Ax^2e^{2x} + B\cos\, 2x + C\sin\, 2x + Dx$
(e) None of the above.

14. A particular solution of $y''' - 5y'' + 6y' = 4e^{-x} - 2$ is:

(a) $z = \frac{1}{3}e^{-x} - \frac{1}{3}x$
(b) $z = -e^{-x} - 2x$
(c) $z = -\frac{1}{3}e^{-x} - 3x$
(d) $z = -\frac{1}{3}e^{-x} - \frac{1}{3}x$
(e) None of the above.

15. The general solution of $y''' - 3y'' + 4y' - 12y = 3xe^{-3x} + 2\sin 2x - 3x$ will have the form:

(a) $y = C_1e^{3x} + C_2\cos\, 2x + C_3\sin\, 2x + (Ax + B)e^{3x} + C\cos 2x + D\sin 2x + Ex$
(b) $y = C_1e^{-3x} + C_2\cos\, 2x + C_3\sin\, 2x + (Ax^2 + Bx)e^{-3x} + Cx\sin 2x + Dx + E$
(c) $y = C_1e^{3x} + C_2\cos\, 2x + C_3\sin\, 2x + (Ax + B)e^{-3x} + Cx\cos 2x + Dx\sin 2x + Ex + F$
(d) $y = C_1e^{-3x} + C_2\cos\, 2x + C_3\sin\, 2x + (Ax^2 + Bx)e^{-3x} + Cx\cos 2x + Dx\sin 2x + Ex + F$
(e) None of the above.

16. A particular solution of $y^{(4)} + 2y''' + 9y'' - 2y' - 10y = 4e^x + (2x - 1)e^{-x} + 2e^x\sin 2x$ will have the form:

(a) $z = Axe^x + (Bx^2 + Cx)e^{-x} + De^x\cos 2x + Ee^x\sin\, 2x$
(b) $z = Ae^x + (Bx + C)e^{-x} + De^x\cos 2x + Ee^x\sin 2x$
(c) $z = Axe^x + (Bx^2 + Cx)e^{-x} + Dxe^x\cos 2x + Exe^x\sin 2x$
(d) $z = Axe^x + (Bx + C)e^{-x} + Dxe^{-x}\cos 2x + Exe^x\sin 2x$
(e) None of the above.