1. $z_1 = 3x+4x^3+xe^{2x}$ and $z_2 = 3x+2x^3+xe^{2x}$ are particular solutions of the linear nonhomogeneous equation $y'' + p(x)y' + q(x)y = f(x)$. $y_1 = x^5$ is a solution of the reduced equation. The general solution of the nonhomogeneous equation is:

   (a) $y = C_1x^5 + C_2x^3 + xe^{2x}$
   (b) $y = C_1x^5 + C_2x + 2x^3 + xe^{2x}$
   (c) $y = C_1x^5 + C_2x^3 + 3x + xe^{2x}$
   (d) $y = C_1x^5 + C_2(2x^3) + C_3(3x + xe^{2x})$
   (e) None of the above.

2. Given the linear nonhomogeneous equation $y'' + p(x)y' + q(x)y = 6x^2$. The functions $y_1(x) = x^{-1},\ y_2(x) = x^3$ are a fundamental set of solutions of the reduced equation. The general solution of the given equation is:

   (a) $y = C_1x^{-1} + C_2x^3 + \frac{6}{5}x^4$
   (b) $y = y = C_1x^{-1} + C_2x^3 + \frac{2}{3}x^4$
   (c) $y = C_1x^{-1} + C_2x^3 - 2x^3$
   (d) $y = C_1x^{-1} + C_2x^3 - x^4$
   (e) None of the above.

3. Find the general solution of

$$y''(x) - \frac{2}{x}y' - \frac{10}{x^2}y = 6x^2 - 2x$$

   Hint: The reduced equation has solutions of the form $y = x^r$.

   (a) $y = C_1x^2 + C_2x^{-5} + x^3 - 5x^4$
   (b) $y = C_1x^{-2} + C_2x^5 + \frac{1}{5}x^3 - x^4$
   (c) $y = C_1x^{-2} + C_2x^5 + x^3 - 5x^4$
   (d) $y = C_1x^{-2} + C_2x^5 + \frac{1}{5}x^2 - x^3$
   (e) None of the above.

4. The general solution of $y'' - 8y' + 16y = \dfrac{e^{4x}}{x}$ is:

   (a) $y = C_1e^{-4x} + C_2xe^{-4x} + e^{4x}\ln x$
   (b) $y = C_1e^{4x} + C_2xe^{4x} - xe^{4x}\ln x$
   (c) $y = C_1e^{4x} + C_2xe^{4x} + xe^{4x}\ln x$
   (d) $y = C_1e^{4x} + C_2e^{-4x} + xe^{4x}\ln x$
   (e) None of the above.

5. The general solution of $y'' + 4y' + 4y = \dfrac{e^{-2x}}{x^2}$ is:

(a) $y = C_1 e^{-2x} + C_2 x e^{-2x} + e^{-2x} \ln x$
(b) $y = C_1 e^{2x} + C_2 x e^{2x} - x e^{2x} \ln x$
(c) $y = C_1 e^{-2x} + C_2 x e^{-2x} - x e^{-2x} \ln x$
(d) $y = C_1 e^{-2x} + C_2 e^{-2x} - e^{-2x} \ln x$
(e) None of the above.

6. The general solution of $y'' + 4y' = 2e^{-4x} + 6$ is:

(a) $y = C_1 e^{-4x} + C_2 - \frac{1}{2} x e^{-4x} + \frac{3}{2} x$
(b) $y = C_1 e^{-4x} + C_2 + \frac{1}{2} x e^{-4x} + \frac{2}{3} x$
(c) $y = C_1 e^{-4x} + C_2 + x e^{-4x} + \frac{3}{2}$
(d) $y = C_1 e^{-4x} + C_2 x - \frac{1}{2} x e^{-4x} + \frac{3}{2}$
(e) None of the above.

7. A particular solution of $y'' - 2y' + 5y = -3e^{x} \cos 2x + 4x$ will have the form:

(a) $y = Axe^{-x} \cos 2x + Bxe^{-x} \sin 2x + Cx$
(b) $y = Axe^{x} \cos 2x + Bxe^{x} \sin 2x + Cx$
(c) $y = Axe^{x} \cos 2x + Bxe^{x} \sin 2x + Cx + D$
(d) $y = Ae^{x} \cos 2x + Be^{x} \sin 2x + Cx + D$
(e) None of the above.

8. A particular solution of $y'' - 2y' - 8y = 3 \sin 4x - 2e^{-2x} + 6$ will have the form:

(a) $y = A \cos 4x + B \sin 4x + Ce^{-2x} + D$
(b) $y = A \cos 4x + B \sin 4x + Cxe^{-2x} + D$
(c) $y = A \cos 4x + B \sin 4x + Cxe^{-2x} + Dx$
(d) $y = A \cos 4x + B \sin 4x + Ce^{-2x} + Dx + E$
(e) None of the above.

9. The general solution of $y^{(4)} - 5y'' - 36y = 0$ is:

(a) $y = C_1 \cos 3x + C_2 \sin 3x + C_3 e^{2x} + C_4 e^{-2x}$
(b) $y = C_1 \cos 3x + C_2 \sin 3x + C_3 \cos 2x + C_4 \sin 2x$
(c) $y = C_1 \cos 2x + C_2 \sin 2x + C_3 e^{3x} + C_4 e^{-3x}$
(d) $y = C_1 \cos 2x + C_2 \sin 2x + C_3 e^{3x} + C_4 x e^{3x}$
(e) None of the above.

10. The general solution of $y^{(4)} - y''' - 3y'' + 17y' - 30y = 0$ is:

(a) $y = C_1e^x \cos 2x + C_2e^x \sin 2x + C_3e^{3x} + C_4e^{-2x}$
(b) $y = C_1 \cos 3x + C_2 \sin 3x + C_3e^{-3x} + C_4e^{2x}$
(c) $y = C_1 \cos 2x + C_2 \sin 2x + C_3e^{-3x} + C_4e^{2x}$
(d) $y = C_1e^x \cos 2x + C_2e^x \sin 2x + C_3e^{-3x} + C_4e^{2x}$
(e) None of the above.

11. The general solution of $y''' + 5y'' + 8y' + 4y = 3e^{-x} + 8x + 4$ is:

(a) $y = C_1e^{-x} + C_2e^{-2x} + C_3xe^{-2x} + 3xe^{-x} + 2x - 3$
(b) $y = C_1e^{-x} + C_2e^{2x} + C_3xe^{2x} - 3xe^{-x} - 2x - 1$
(c) $y = C_1e^{x} + C_2e^{-2x} + C_3xe^{-2x} - 2xe^{-x} + 4x - 3$
(d) $y = C_1e^{x} + C_2e^{2x} + C_3e^{-2x} + 3xe^{-x} - 2x + 3$
(e) None of the above.

12. A particular solution of $y^{(4)} - 3y'' - 4y = 2e^{2x} - e^{-x} + 4\cos x$ will have the form:

(a) $z = Axe^{2x} + Be^{-x} + Cx \cos x + Dx \sin x$
(b) $z = Ae^{2x} + Bxe^{-x} + C \cos x + D \sin x$
(c) $z = Ax^2e^{2x} + Bxe^{-x} + Cx \cos x + Dx \sin x$
(d) $z = Axe^{2x} + Be^{-x} + C \cos x + D \sin x$
(e) None of the above.

13. If $f(x) = 5 + 2x^2 + 3e^{2x} - 2xe^{-x} + e^x \cos 2x$, then $F(s) =$

(a) $\frac{5}{s} + \frac{2}{s^3} + \frac{3}{s-2} - \frac{1}{(s+1)^2} + \frac{2}{s^2 - 2s + 5}$
(b) $\frac{5}{s} + \frac{4}{s^2} + \frac{3}{s+2} - \frac{2}{(s-1)^2} + \frac{s-1}{s^2 - 2s + 5}$
(c) $\frac{5}{s} + \frac{4}{s^3} + \frac{3}{s-2} - \frac{2}{(s+1)^2} + \frac{s-1}{s^2 - 2s + 5}$
(d) $\frac{5}{s} + \frac{4}{s^3} + \frac{3}{s+2} - \frac{2}{(s+1)^2} + \frac{s+1}{s^2 - 2s + 5}$
(e) None of the above.

14. The Laplace transform of the solution of $y'' - 7y' + 10y = 3x + 2e^{2x}$, $y(0) = 2$, $y'(0) = -2$ is:

(a) $Y = \frac{2s^2 + 3s - 6}{s^2(s-2)(s^2 - 7s + 10)} + \frac{2s + 16}{s^2 - 7s + 10}$
(b) $Y = \frac{2s^2 + 3s - 6}{s^2(s-2)(s^2 - 7s + 10)} + \frac{2s - 16}{s^2 - 7s + 10}$

(c) $Y = \dfrac{2s^2 - 3s + 6}{s^2(s-2)(s^2 - 7s + 10)} - \dfrac{16}{s^2 - 7s + 10}$

(d) $Y = \dfrac{2s^2 + 3s - 6}{s^2(s-2)(s^2 - 7s + 10)} + \dfrac{16}{s^2 - 7s + 10}$

(e) None of the above.

15. If $F(s) = \dfrac{s^2 - 2s + 4}{(s-4)(s^2+4)}$, then $f(x) =$

(a) $f(x) = \frac{3}{5}e^{4x} - \frac{3}{5}\cos 2x + \frac{1}{5}\sin 2x$
(b) $f(x) = \frac{2}{5}e^{4x} + \frac{3}{5}\cos 2x - \frac{3}{10}\sin 2x$
(c) $f(x) = \frac{3}{5}e^{4x} + \frac{2}{5}\cos 2x - \frac{1}{5}\sin 2x$
(d) $f(x) = \frac{2}{5}e^{4x} + \frac{2}{5}\cos 2x - \frac{2}{5}\sin 2x$
(e) None of the above.

16. If $F(s) = \dfrac{s^2 + 2s - 2}{(s+2)(s^2 - 4s + 8)}$, then $f(x) =$

(a) $f(x) = \frac{1}{10}e^{-2x} - \frac{11}{10}e^{2x}\cos 2x - \frac{4}{5}e^{2x}\sin 2x$
(b) $f(x) = \frac{1}{10}e^{-2x} + \frac{11}{10}e^{2x}\cos 2x + \frac{8}{5}e^{2x}\sin 2x$
(c) $f(x) = -\frac{1}{10}e^{-2x} + \frac{11}{10}e^{2x}\cos 2x + \frac{8}{5}e^{2x}\sin 2x$
(d) $f(x) = -\frac{1}{10}e^{-2x} + \frac{11}{10}e^{2x}\cos 2x + \frac{4}{5}e^{2x}\sin 2x$
(e) None of the above.

17. If $f(x) = \begin{cases} x^2 - 1, & 0 \le x < 3 \\ 2x + 3, & x \ge 3 \end{cases}$, then $\mathcal{L}[f(x)] =$.

(a) $\mathcal{L}[f(x)] = \dfrac{2}{s^3} - \dfrac{1}{s} - e^{-3s}\dfrac{2}{s^3} - 4e^{-3s}\dfrac{1}{s^2} + e^{-3s}\dfrac{1}{s}$

(b) $\mathcal{L}[f(x)] = \dfrac{2}{s^3} - \dfrac{1}{s} + e^{-3s}\dfrac{2}{s^3} + e^{-3s}\dfrac{5}{s^2} + e^{-3s}\dfrac{3}{s}$

(c) $\mathcal{L}[f(x)] = \dfrac{2}{s^3} - \dfrac{1}{s} - e^{-3s}\dfrac{2}{s^3} + 3e^{-3s}\dfrac{1}{s^2} + e^{-3s}\dfrac{1}{s}$

(d) $\mathcal{L}[f(x)] = \dfrac{2}{s^3} - \dfrac{1}{s} + e^{-3s}\dfrac{2}{s^3} - 2e^{-3s}\dfrac{1}{s^2} + e^{-3s}\dfrac{6}{s}$

(e) None of the above.

18. If $f(x) = \begin{cases} 3x - 2, & 0 \le x < 2 \\ 2e^{4x}, & x \ge 2 \end{cases}$, then $\mathcal{L}[f(x)] =$.

(a) $\mathcal{L}[f(x)] = \dfrac{3}{s^2} - \dfrac{2}{s} + e^{-2s}\left[\dfrac{3}{s^2} - \dfrac{2}{s} - \dfrac{2e^4}{s-4}\right]$

(b) $\mathcal{L}[f(x)] = \dfrac{3}{s^2} - \dfrac{2}{s} - e^{-2s}\left[\dfrac{3}{s^2} - \dfrac{4}{s} + \dfrac{2e^8}{s-4}\right]$

(c) $\mathcal{L}[f(x)] = \dfrac{3}{s^2} - \dfrac{2}{s} + e^{-2s}\left[\dfrac{3}{s^2} + \dfrac{6}{s} - \dfrac{2e^4}{s-4}\right]$

(d) $\mathcal{L}[f(x)] = \dfrac{3}{s^2} - \dfrac{2}{s} - e^{-2s}\left[\dfrac{3}{s^2} + \dfrac{4}{s} - \dfrac{2e^8}{s-4}\right]$

(e) None of the above.

19. If $f(x) = \begin{cases} \sin 3x, & 0 \le x < \pi \\ 2\cos x, & x \ge \pi \end{cases}$, then $\mathcal{L}[f(x)] =$

(a) $\mathcal{L}[f(x)] = \dfrac{3}{s^2+9} - e^{-\pi s}\left[\dfrac{3}{s^2+9} - \dfrac{2s}{s^2+1}\right]$

(b) $\mathcal{L}[f(x)] = \dfrac{3}{s^2+9} + e^{-\pi s}\left[\dfrac{3}{s^2+9} - \dfrac{2s}{s^2+1}\right]$

(c) $\mathcal{L}[f(x)] = \dfrac{3}{s^2+9} + e^{-\pi s}\left[\dfrac{3}{s^2+9} - \dfrac{2s}{s^2+9}\right]$

(d) $\mathcal{L}[f(x)] = \dfrac{3}{s^2+9} - e^{-\pi s}\left[\dfrac{3}{s^2+9} + \dfrac{2s}{s^2+1}\right]$

(e) None of the above.

20. If $F(s) = \dfrac{2}{s} + \dfrac{4}{s^3} + e^{-2s}\left(\dfrac{3}{s} - \dfrac{2}{s^2} - \dfrac{5}{(s-3)^2}\right)$, then $\mathcal{L}^{-1}[F(s)] =$

(a) $f(x) = \begin{cases} 2 + x^2, & 0 \le x < 2 \\ 9 - 2x + x^2 - 5(x-2)e^{-3(x-2)}, & x \ge 2 \end{cases}$

(b) $f(x) = \begin{cases} 2 + 2x^2, & 0 \le x < 2 \\ 5 - 2x + 2x^2 - 5xe^{3x}, & x \ge 2 \end{cases}$

(c) $f(x) = \begin{cases} 2 + 2x^2, & 0 \le x < 2 \\ 9 - 2x + 2x^2 - 5(x-2)e^{3(x-2)}, & x \ge 2 \end{cases}$

(d) $f(x) = \begin{cases} 2 + 4x^2, & 0 \le x < 2 \\ 5 - 2x + 4x^2 - 5xe^{3x-6}, & x \ge 2 \end{cases}$

(e) None of the above.

21. If $F(s) = \dfrac{2s^2+3s+2}{(s-2)(s^2+4)} + \dfrac{(2s+5)e^{-3s}}{s^2+9}$, then $\mathcal{L}^{-1}[F(s)] =$

(a) $f(x) = \begin{cases} 2e^{2x} + \frac{3}{2}\sin\,2x, & 0 \le x < 3 \\ 2\cos\,3(x-3) + \frac{5}{3}\sin\,3(x-3), & x \ge 3 \end{cases}$

(b) $f(x) = \begin{cases} 2e^{2x} + 3\cos\,2x, & 0 \le x < 3 \\ 2e^{2x} + 3\cos\,2x + \cos 3x + \frac{5}{3}\sin 3x, & x \ge 3 \end{cases}$

(c) $f(x) = \begin{cases} 2e^{2x} + \frac{3}{2}\sin\,2x, & 0 \le x < 3 \\ 2e^{2x} + \frac{3}{2}\sin 2x + 2\cos 3(x-3) + \frac{5}{3}\sin 3(x-3), & x \ge 3 \end{cases}$

(d) $f(x) = \begin{cases} 2e^{2x} + 3\sin 2x, & 0 \le x < 3 \\ 2e^{2x} + 3\sin 2x + 2\cos(3x-9) + 5\sin(3x-9), & x \ge 3 \end{cases}$

(e) None of the above.