1. If the rank of the augmented matrix of a system of n linear equations in n unknowns is greater than the rank of the matrix of coefficients, then the system is inconsistent.

   (a) Always true.

   (b) Sometimes true.

   (c) Never true.

   (d) None of the above.

2. If a system of n linear equations in n unknowns has infinitely many solutions, then the rank of the matrix of coefficients is $n - 1$.

   (a) Always true.

   (b) Sometimes true.

   (c) Never true.

   (d) None of the above.

3. If the rank of the augmented matrix of a system of n linear equations in n unknowns equals the rank of the matrix of coefficients, then the system has a unique solution.

   (a) Always true.

   (b) Sometimes true.

   (c) Never true.

   (d) None of the above.

$$2x + 8y + 11z = 7$$

4. The solution set of the system $x + 4y + 3z = 1$ is:

$$x + 6y + 7z = 3$$

   (a) $x = 1,\ y = -1,\ z = 2$

   (b) no solution

   (c) $x = 2 - a,\ y = -3 - 2a,\ z = a,$ a any real number

   (d) $x = 2,\ y = -1,\ z = 1$

   (e) None of the above.

$$3x + 6y - 3z = 6$$

5. The solution set of the system $-2x - 4y - 3z = -1$ is:

$$3x + 6y - 2z = 10$$

(a) $x = 7 - a$, $y = 2 + a$, $z = a$, a any real number.

(b) $x = 3 - 2a$, $y = 4a - 1$, $z = a$, a any real number.

(c) no solution

(d) $x = 4$, $y = -2$, $z = -1$.

(e) None of the above.

6. The solution set of the system
$$\begin{aligned} x + 2y - z &= 3 \\ 2x + 4y - 2z &= 6 \\ 3x + 6y + 2z &= -1 \end{aligned}$$
  is:

(a) $x = 1 - 2a$, $y = a$, $z = -2$, a any real number.

(b) $x = 3 - 2a$, $y = 4a - 1$, $z = a$, a any real number.

(c) $x = 2 - 2a$, $y = a$, $z = -1$, a any real number.

(d) $x = -1$, $y = -2$, $z = -2$.

(e) None of the above.

7. The solution set of the system
$$\begin{aligned} x + 2y - 3z + 4w &= 2 \\ 2x + 5y - 2z + w &= 1 \\ 5x + 12y - 7z + 7w &= 4 \end{aligned}$$
  is:

(a) no solutions

(b) $x = -8 + 11a$, $y = 2 - 4a$, $z = a$, $w = a$, a any real number

(c) $x = 8 + 11a$, $y = -3 - 4a$, $z = a$, $w = 0$, a any real number

(d) $x = 8 + a$, $y = -3 + 4a$, $z = 1$, $w = a$, a any real number

(e) None of the above.

8. The solution set of the system
$$\begin{aligned} 2y - 3z + w &= 0 \\ x + y - z + 4w &= 0 \\ -2x - 2y + 2z - 8w &= 0 \end{aligned}$$
  is:

(a) no solutions

(b) $x = -\frac{1}{2}a - \frac{7}{2}b$, $y = \frac{3}{2}a - \frac{1}{2}b$, $z = a$, $w = b$, a, b any real numbers

(c) $x = \frac{1}{2}a + \frac{7}{2}b$, $y = -\frac{3}{2}a + \frac{1}{2}b$, $z = a$, $w = b$, a, b any real numbers

(d) $x = -\frac{1}{2}a - \frac{7}{2}b$, $y = \frac{3}{2}a - \frac{1}{2}b$, $z = a$, $w = 0$, a any real number

(e) None of the above.

9. The solution set of the system
$$x + y - 2z + 3w = 4$$
$$2x + 3y + 3z - w = 3$$
$$5x + 7y + 4z + w = 5$$
is:

    (a) no solutions

    (b) $x = 9 + 9a - 10b$, $y = -5 - 7a + 7w$, $z = a$, $w = b$, a, b any real numbers

    (c) $x = 8 + 11a$, $y = -3 - 4a$, $z = a$, $w = 0$, a any real number

    (d) $x = 9 + 9a$, $y = -5 - 7a$, $z = a$, $w = 0$, a any real numbers

    (e) None of the above.

Use the system of equations
$$x - 2y = 1$$
$$x - y + kz = -1$$
$$ky + 9z = 6$$
for problems 10 and 11.

10. The value(s) of k such that the system has a unique solution is (are):

    (a) $k \neq 3$

    (b) $k \neq \pm 3$

    (c) $k \neq -3$

    (d) $k = 3, -3$

    (e) None of the above.

11. The value(s) of k such that the system has a infinitely many solutions is (are):

    (a) $k = -3$

    (b) $k = 3$

    (c) $k = -2$

    (d) $k \neq -3$

    (e) None of the above.

Use the system of equations
$$x - 4y - 2z = 2$$
$$y + 2z = 2$$
$$-2x + 7y + (k^2 - 2)z = k - 4$$
for problems 12 and 13.

12. The value(s) of k such that the system has a unique solution is (are):

    (a) $k \neq 2$

    (b) $k = -1, 2$

    (c) $k \neq \pm 2$

    (d) $k \neq -2$

    (e) None of the above.

13. The value(s) of k such that the system has no solution is (are):

   (a) $k = 2, -3$
   (b) $k = 0$
   (c) $k = 2$
   (d) $k = -2$
   (e) None of the above.

14. The ranks of the augmented matrix and the matrix of coefficients of the system of equations

$$
\begin{aligned}
x + 2y - 3z &= 1 \\
2x + 5y - 8z &= 4 \\
3x + 8y - 13z &= 7
\end{aligned}
$$

   are, respectively,

   (a) 3, 3
   (b) 2, 3
   (c) 2, 2
   (d) 3, 2
   (e) None of the above.

15. The ranks of the augmented matrix and the matrix of coefficients of the system of equations

$$
\begin{aligned}
x + 2y - 3z &= -1 \\
3x - y + 2z &= 7 \\
5x + 3y - 4z &= 2
\end{aligned}
$$

   are, respectively,

   (a) 3, 3
   (b) 2, 3
   (c) 2, 2
   (d) 3, 2
   (e) None of the above.

16. The system of equations in Problem in 15 has

   (a) a unique solution.
   (b) infinitely many solutions.
   (c) no solution.
   (d) All of the above.
   (e) None of the above.

17. The system of equations

$$x + 2y = 3$$
$$2x + 5y - z = -4$$
$$3x - 2y - z = 5$$

is:

(a) consistent and independent.

(b) consistent and dependent.

(c) inconsistent.

(d) All of the above.

(e) None of the above.

18. The system of equations

$$x + 2y - 2z = -1$$
$$3x - y + 2z = 7$$
$$5x + 3y - 2z = 2$$

is:

(a) consistent and independent.

(b) consistent and dependent.

(c) inconsistent.

(d) incoherent.

(e) None of the above.