1. If the matrix of coefficients of a homogeneous system of n linear equations in n unknowns does not have an inverse, then the system has infinitely many solutions.

   (a) Always true
   (b) Sometimes true
   (c) Never true, i.e., false
   (d) None of the above

2. If the matrix of coefficients of a system of n linear equations in n unknowns is singular, then the system does not have a unique solution.

   (a) Always true
   (b) Sometimes true
   (c) Never true, i.e., false
   (d) None of the above

3. If a system of n linear equations in n unknowns is inconsistent, then the rank of the matrix of coefficients is less than or equal to $n - 1$.

   (a) Always true
   (b) Sometimes true
   (c) Never true, i.e., false
   (d) None of the above

4. If the rank of the matrix of coefficients of a system of m linear equations in n unknowns equals the rank of the augmented matrix, then the system has infinitely many solutions.

   (a) Always true
   (b) Sometimes true
   (c) Never true, i.e., false
   (d) None of the above

5. If a, b, and c are integers, and $a \neq 0$, then $\begin{pmatrix} a & b \\ c & \sqrt{2} \end{pmatrix}$ is nonsingular.

   (a) Always true.
   (b) Almost always true (i.e., true with probability 1)
   (c) Sometimes true.

(d) Never true.

(e) None of the above.

6. The real numbers a for which that the vectors

$$\mathbf{v}_1 = \begin{pmatrix} a \\ 2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 8 \\ a \end{pmatrix}$$

are linearly dependent are:

(a) $a = \pm 4$

(b) $a \neq \pm 4$

(c) The vectors are linearly dependent for all real numbers a.

(d) $a = 4$

(e) The vectors are linearly independent for all real numbers a.

7. The real numbers a for which that the vectors

$$\mathbf{v}_1 = (a, -9), \quad \mathbf{v}_2 = (-4, a), \quad \mathbf{v}_3 = (-2, 5)$$

are linearly independent are:

(a) $a = 4, 9$

(b) $a \neq 4, 9$

(c) The vectors are linearly dependent for all real numbers a.

(d) $a = -4, -9$

(e) The vectors are linearly independent for all real numbers a.

8. The real numbers a for which that the vectors

$$\mathbf{v}_1 = \begin{pmatrix} a \\ 0 \\ -2 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 4 \\ a \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} -1 \\ 2 \\ a \end{pmatrix}$$

are linearly independent are:

(a) $a = \pm 2$

(b) $a = -4$

(c) The vectors are linearly independent for all real numbers a.

(d) $a \neq \pm 2$

(e) The vectors are linearly dependent for all real numbers a.

9. The real number(s) a for which that the vectors

$$\mathbf{v}_1 = (-1, 1, 3), \quad \mathbf{v}_2 = (a, 5, 2), \quad \mathbf{v}_3 = (4, -3, 2), \quad \mathbf{v}_4 = (2, a, -1)$$

are linearly independent is (are):

(a) $a \neq 1, -4$

(b) $a =\neq \pm 2$

(c) The vectors are linearly independent for all real numbers a.

(d) $a \neq -2, 4, 1$

(e) The vectors are linearly dependent for all real numbers a.

10. The value(s) of x such that $A = \begin{pmatrix} 2 & -1 & 4 \\ x & 0 & 2 \\ 0 & -1 & x \end{pmatrix}$ is nonsingular is (are)

(a) $= 1, -2$

(b) $x \neq \pm 2$

(c) $x = 2$

(d) $x \neq 2$

(e) A is nonsingular for all real numbers x.

11. The values of λ such that the rows of $\begin{pmatrix} -5 & 1 & 3 \\ 0 & 1 & \lambda \\ \lambda & 0 & 2 \end{pmatrix}$ are linearly dependent are:

(a) $\lambda \neq -5,\ 2$

(b) $\lambda = 2,\ -5$

(c) $\lambda \neq 5,\ -2$

(d) $\lambda = 5,\ -2$

(e) The rows are linearly dependent for all real numbers λ.

12. Set $A = \begin{pmatrix} 2 & 5 & -3 & -2 \\ 0 & 3 & -2 & -1 \\ 1 & 3 & -2 & 2 \\ -1 & -6 & 4 & 3 \end{pmatrix}$. Then, $\det A =$

(a) -4

(b) 7

(c) -10

(d) 13

(e) 2

13. The maximum number of independent vectors in the set

$$\{\mathbf{v}_1 = (\,2,\ 0,\ -1\,),\ \mathbf{v}_2 = (\,-3,\ 1,\ 2\,),\ \mathbf{v}_3 = (\,8,\ -2,\ -5\,),\ \mathbf{v}_4 = (\,-9,\ 1,\ 5\,)\}$$

is:

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

14. The maximum number of independent vectors in the set

$$\mathbf{v}_1 = (1, -1, 2, 1),\ \mathbf{v}_2 = (3, 2, 0, -1),\ \mathbf{v}_3 = (-1, -4, 4, 3),\ \mathbf{v}_4 = (2, 3, -4, -1)$$

is:

(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

15. Given the matrix $A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$. If n is the rank of A, then which of the following is not true:

(a) $n \geq 1$

(b) $n = 2$

(c) $n \leq 4$

(d) $n \neq 3.2$

(e) None of the above