1. If the rank of the augmented matrix of a system of n linear equations in n unknowns equals the rank of the matrix of coefficients, then the system has a unique solution.
   (a) Always true.
   (b) Sometimes true.
   (c) Never true.
   (d) None of the above.

2. If 0 is an eigenvalue of the matrix of coefficients of a system of n linear equations in n unknowns, then the system has infinitely many solutions.
   (a) Always true.
   (b) Sometimes true.
   (c) Never true.
   (d) None of the above.

3. If a system of n linear equations in n unknowns is dependent, then 0 is an eigenvalue of the matrix of coefficients.
   (a) Always true.
   (b) Sometimes true.
   (c) Never true.
   (d) None of the above.

4. If the reduced row echelon form of the matrix of coefficients of a system of n linear equations in n unknowns is I_n, then the matrix of coefficients is singular.
   (a) Always true.
   (b) Sometimes true.
   (c) Never true.
   (d) None of the above.

5. The eigenvalues of $\begin{pmatrix} -1 & 6 \\ 2 & 3 \end{pmatrix}$ are:
   (a) $\{5, -2\}$
   (b) $\{5, -3\}$
   (c) $\{4, -3\}$
   (d) $\{-5, 3\}$
   (e) None of the above.

6. The eigenvalues of $\begin{pmatrix} -2 & -8 \\ 2 & -2 \end{pmatrix}$ are:

(a) $\{2+4i, 2-4i\}$
(b) $\{5, 4\}$
(c) $\{2, 10\}$
(d) $\{-2+4i, -2-4i\}$
(e) None of the above.

7. The eigenvalues of $\begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ are:

(a) $\{3\}$
(b) $\{3, -3\}$
(c) $\{-3\}$
(d) $\{2, 4\}$
(e) None of the above.

8. Which of the following are eigenvectors for $\begin{pmatrix} 3 & 1 \\ -1 & 5 \end{pmatrix}$?

(a) $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$
(b) $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$
(c) $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$
(d) $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$
(e) None of the above.

9. Which of the following are eigenvectors for $\begin{pmatrix} 3 & -10 \\ 1 & 1 \end{pmatrix}$?

(a) $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} - i \begin{pmatrix} -3 \\ 0 \end{pmatrix} \right\}$
(b) $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} + i \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \end{pmatrix} - i \begin{pmatrix} 0 \\ 3 \end{pmatrix} \right\}$
(c) $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} + i \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} - i \begin{pmatrix} 3 \\ 0 \end{pmatrix} \right\}$
(d) $\left\{ \begin{pmatrix} 1 \\ -1 \end{pmatrix} + i \begin{pmatrix} 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} - i \begin{pmatrix} 3 \\ 0 \end{pmatrix} \right\}$
(e) None of the above.

10. The eigenvalues of a 3×3 matrix A are $\lambda_1 = 3,\ \lambda_2 = 2.\ \lambda_3 = -2$. The characteristic equation of A is:

(a) $\lambda^3 + 3\lambda^2 - 4\lambda - 12 = 0$
(b) $\lambda^3 - 2\lambda^2 + 3\lambda - 10 = 0$
(c) $(\lambda + 3)(\lambda + 2)(\lambda - 2) = 0$
(d) $\lambda^3 - 3\lambda^2 - 4\lambda + 12 = 0$
(e) None of the above.

11. The eigenvalues of $\begin{pmatrix} 3 & -3 & 5 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{pmatrix}$ are:

(a) $\{2, 3\}$
(b) $\{-2, 3\}$
(c) $\{3, -3, -2\}$
(d) $\{3, -2, 2\}$
(e) None of the above.

12. The characteristic polynomial of $A = \begin{pmatrix} 5 & -2 & 2 \\ 4 & -3 & 4 \\ 4 & -6 & 7 \end{pmatrix}$ is: (Hint: 5 is an eigenvalue.)

(a) $-(\lambda - 5)^2(\lambda - 3)$
(b) $-(\lambda - 5)(\lambda - 3)(\lambda - 1)$
(c) $-(\lambda + 3)(\lambda + 4)(\lambda + 1)$
(d) $-(\lambda - 5)(\lambda + 3)(\lambda - 1)$
(e) None of the above.

13. The number of independent eigenvectors of $\begin{pmatrix} 2 & 2 & 1 \\ 0 & 4 & 0 \\ 4 & -2 & -1 \end{pmatrix}$ is:

(a) 1, (b) 2, (c) 3, (d) 4, (e) None of the above.

14. The eigenvalues of $\begin{pmatrix} 6 & -2 & -4 \\ -4 & 8 & 8 \\ 4 & -4 & -4 \end{pmatrix}$ are: $\lambda_1 = 2,\ \lambda_2 = \lambda_3 = 4$. The number of independent eigenvectors is:

(a) 1, (b) 2, (c) 3, (d) 4, (e) None of the above.

15. The eigenvalues of $\begin{pmatrix} 0 & 6 & -5 \\ 1 & 4 & -4 \\ 2 & 10 & -9 \end{pmatrix}$ are: $\lambda_1 = \lambda_2 = -2,\ \lambda_3 = -1$. The number of independent eigenvectors is:

(a) 1, (b) 2, (c) 3, (d) 4, (e) None of the above.

16. The eigenvalues of $\begin{pmatrix} 1 & 3 & 3 \\ 4 & 2 & 4 \\ -2 & -2 & -4 \end{pmatrix}$ are: $\lambda_1 = 3,\ \lambda_2 = \lambda_3 = -2$. Which of the following is not an eigenvector: (a) $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$ (b) $\begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$ (c) $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ (d) $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(e) Each of these is an eigenvector.

17. The eigenvalues of $\begin{pmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{pmatrix}$ are: $\lambda_1 = 4,\ \lambda_2 = \lambda_3 = -2$. Which of the following is not an eigenvector:

(a) $\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ (d) $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

(e) Each of these is an eigenvector.