1. If the rank of the augmented matrix of a system of n linear equations in n unknowns is greater than the rank of the matrix of coefficients, then 0 is an eigenvalue of the matrix of coefficients.

   (a) Always true.

   (b) Sometimes true.

   (c) Never true.

   (d) None of the above.

2. If 0 is an eigenvalue of the matrix of coefficients of a homogeneous system of n linear equations in n unknowns, then the trivial solution is the only solution of the system.

   (a) Always true.

   (b) Sometimes true.

   (c) Never true.

   (d) None of the above.

3. If the matrix of coefficients of a system of n linear equations in n unknowns does not have an inverse, then the system has infinitely many solutions.

   (a) Always true

   (b) Sometimes true

   (c) Never true, i.e., false

   (d) None of the above

4. If the matrix of coefficients of a system of n linear equations in n unknowns is singular, then the system does not have a unique solution.

   (a) Always true

   (b) Sometimes true

   (c) Never true, i.e., false

   (d) None of the above

In Problems 5 - 8, the given system is equivalent to a linear homogenous differential equation with constant coefficients.

5. A fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ 15 & 2 \end{pmatrix}\mathbf{x}$ is:

(a) $\left\{ e^{-5t}\begin{pmatrix} 1 \\ 5 \end{pmatrix}, e^{3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\}$

(b) $\left\{ e^{5t}\begin{pmatrix} -1 \\ 5 \end{pmatrix}, e^{-3t}\begin{pmatrix} 1 \\ 3 \end{pmatrix} \right\}$

(c) $\left\{ e^{5t}\begin{pmatrix} 1 \\ 5 \end{pmatrix}, e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} \right\}$

(d) $\left\{ e^{5t}\begin{pmatrix} 1 \\ 5 \end{pmatrix}, e^{-3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix} \right\}$

(e) None of the above.

6. The general solution of $\mathbf{x}' = \begin{pmatrix} 0 & 1 \\ -8 & 6 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x} = C_1 e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{-4t}\begin{pmatrix} 1 \\ -4 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 2 \\ 1 \end{pmatrix} + C_2 e^{4t}\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix} + C_2 e^{4t}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix} + C_2 e^{4t}\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

(e) None of the above.

7. The general solution of $\mathbf{x}' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + C_3 e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + C_2 e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} + C_3 e^{t}\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$

(e) None of the above.

8. A fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & 1 & 2 \end{pmatrix}\mathbf{x}$ is:

(a) $\left\{ e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix},\ e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\ e^{-t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

(b) $\left\{ e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix},\ e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\ e^{-t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

(c) $\left\{ e^{2t}\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix},\ e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix},\ e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

(d) $\left\{ e^{-2t}\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix},\ e^{t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix},\ e^{t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right\}$

(e) None of the above.

9. The general solution of $\mathbf{x}' = \begin{pmatrix} -6 & -3 \\ 1 & -2 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x} = C_1 e^{-5t}\begin{pmatrix} 3 \\ 1 \end{pmatrix} + C_2 e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{5t}\begin{pmatrix} 3 \\ 1 \end{pmatrix} + C_2 e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{-4t}\begin{pmatrix} 1 \\ -4 \end{pmatrix} + C_2 e^{-3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{-5t}\begin{pmatrix} -3 \\ 1 \end{pmatrix} + C_2 e^{-3t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(e) None of the above.

10. A fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ 1 & -2 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x}_1 = e^{3t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

(b) $\mathbf{x}_1 = e^{-4t}\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{-t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) $\mathbf{x}_1 = e^{-4t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(d) $\mathbf{x}_1 = e^{-3t}\begin{pmatrix} 1 \\ -3 \end{pmatrix}, \quad \mathbf{x}_2 = e^{-2t}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

(e) None of the above.

11. The general solution of $\mathbf{x}' = \begin{pmatrix} 1 & 5 \\ 3 & 3 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x}_1 = e^{-6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{2t}\begin{pmatrix} 5 \\ -3 \end{pmatrix}$

(b) $\mathbf{x}_1 = e^{4t}\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(c) $\mathbf{x}_1 = e^{-4t}\begin{pmatrix} 2 \\ -1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{-t}\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(d) $\mathbf{x}_1 = e^{6t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{-2t}\begin{pmatrix} -5 \\ 3 \end{pmatrix}$

(e) None of the above.

12. The general solution of $\mathbf{x}' = \begin{pmatrix} -2 & 2 & 1 \\ 0 & -1 & 0 \\ 2 & -2 & -1 \end{pmatrix}\mathbf{x}$ is:

(a) $\mathbf{x} = C_1 e^{-3t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + C_2 e^{-t}\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} + C_3\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{3t}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + C_2 e^{-t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_3 e^{t}\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{-3t}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + C_2 e^{-t}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{3t}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + C_2 e^{-t}\begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix} + C_3\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

(e) None of the above.

13. The eigenvalues of $A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$ are: $\lambda_1 = 4,\ \lambda_2 = 1,\ \lambda_3 = -1$. The general solution of $\mathbf{x}' = A\,\mathbf{x}$ is:

(a) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_2 e^{t}\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} + C_3 e^{-t}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

(e) None of the above.

14. The characteristic equation of $A = \begin{pmatrix} 1 & -3 & 1 \\ -1 & 1 & 1 \\ 3 & -3 & -1 \end{pmatrix}\mathbf{x}$ is $\lambda^3 - \lambda^2 - 4\lambda + 4 = 0$. The general solution of $\mathbf{x}' = A\mathbf{x}$ is:

(a) $\mathbf{x} = C_1\, e^{-2t}\begin{pmatrix} 3 \\ -2 \\ -3 \end{pmatrix} + C_2\, e^{2t}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_3\, e^{t}\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

(b) $\mathbf{x} = C_1\, e^{-2t}\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} + C_2\, e^{2t}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + C_3\, e^{t}\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

(c) $\mathbf{x} = C_1\, e^{-2t}\begin{pmatrix} 3 \\ -2 \\ -3 \end{pmatrix} + C_2\, e^{2t}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + C_3\, e^{t}\begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix}$

(d) $\mathbf{x} = C_1\, e^{-2t}\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} + C_2\, e^{2t}\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + C_3\, e^{t}\begin{pmatrix} 3 \\ -1 \\ 3 \end{pmatrix}$

(e) None of the above.

15. The characteristic equation of $A = \begin{pmatrix} 1 & -1 & 2 \\ 12 & -4 & 10 \\ -6 & 1 & -7 \end{pmatrix}\mathbf{x}$ is $\lambda^3 + 10\lambda^2 + 31\lambda + 30 = 0$. A fundamental set of solutions of $\mathbf{x}' = A\mathbf{x}$ is:

(a) $\left\{ e^{2t}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix},\ e^{-3t}\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix},\ e^{-5t}\begin{pmatrix} -2 \\ 3 \\ 3 \end{pmatrix} \right\}$

(b) $\left\{ e^{-2t}\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix},\ e^{-3t}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix},\ e^{5t}\begin{pmatrix} -2 \\ -4 \\ 3 \end{pmatrix} \right\}$

(c) $\left\{ e^{-2t}\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix},\ e^{-3t}\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix},\ e^{-5t}\begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} \right\}$

(d) $\left\{ e^{-2t}\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix},\ e^{3t}\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix},\ e^{5t}\begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} \right\}$

(e) None of the above.

16. The the eigenvalues of $A = \begin{pmatrix} -2 & 2 & 6 \\ 2 & 6 & 2 \\ -2 & -2 & 2 \end{pmatrix}$ are $\lambda_1 = 4, \lambda_2 = 2, \lambda_3 = 0$. Find the general solution of $\mathbf{x}' = A\mathbf{x}$.

(a) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

(b) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} -1 \\ -3 \\ 2 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + C_3\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

(c) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + C_3\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

(d) $\mathbf{x} = C_1 e^{4t}\begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + C_2 e^{2t}\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} + C_3\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

(e) None of the above.