1. The general solution of $x^2y' = 4x^3 - 2xy$ is:

   (a) $y = \frac{2}{3}x^3 + Cx^{-2}$
   (b) $y = x^2 + Cx^{-2}$
   (c) $y = 2 + Cx^{-2}$
   (d) $y = x^6 + Cx^2$
   (e) None of the above.

2. The general solution of $x^2y' = 3x^5e^{2x} + 3xy$ is:

   (a) $y = \frac{1}{3}x^3e^{2x} + C$
   (b) $y = 3x^3e^{2x} + Cx^3$
   (c) $y = \frac{3}{2}x^3e^{2x} + C$
   (d) $y = \frac{3}{2}x^3e^{2x} + Cx^3$
   (e) None of the above.

3. The general solution of $y' = 2xy + 4xe^{x^2}$ is

   (a) $y = 2x^2e^{x^2} + Ce^{x^2}$
   (b) $y = 2x^2e^{-x^2} + Ce^{-x^2}$
   (c) $y = 4xe^{-x^2} + Ce^{x^2}$
   (d) $y = 2x^2e^{x^2} + C$
   (e) None of the above.

4. The general solution of $\dfrac{dy}{dx} = \dfrac{xy^2 - x}{y}$ is

   (a) $y^2 = Ce^{x^2} - 1$
   (b) $\ln|y^2 - 1| = \ln x + C$
   (c) $\ln|y^2 - 1| = Ce^{x^2}$
   (d) $y^2 = Ce^{x^2} + 1$
   (e) None of the above.

5. The general solution of $e^x \dfrac{dy}{dx} = e^{-y} + e^{2x-y}$ is:

(a) $e^y = C + e^{-x} + \frac{1}{2}\, e^{2x}$

(b) $y = \ln\left(e^x - e^{-x} + C\right)$

(c) $y = \ln\left(e^x + e^{-x} + C\right)$

(d) $e^y = C - e^{-x} - \frac{1}{2}\, e^{-2x}$

(e) None of the above.

6. The solution of the initial-value problem $y' + y = \dfrac{1}{1+e^x}, \quad y(0) = \ln 6$ is

(a) $y = \dfrac{\ln(1+e^x)}{e^x} + \dfrac{\ln 3}{e^x}$

(b) $y = \dfrac{2 \ln 3}{1+e^x}$

(c) $y = \ln\left(\dfrac{e^x}{1+e^x}\right) + \dfrac{\ln 3}{e^x}$

(d) $y = \dfrac{e^x}{1+e^x} - \ln 3$

(e) None of the above.

7. The general solution of $y' + \tan(x) y = 2\cos^2 x$ is:

(a) $y = \frac{1}{2} \sin^2 x + C \cos\, x$

(b) $y = \sin\, x + \dfrac{C}{\cos\, x}$

(c) $y = \sin 2x + C \cos\, x$

(d) $y = \sin x + C \cos x$

(e) None of the above.

8. The general solution of $2y \ln(x)\, y' = \dfrac{y^2+1}{x}$ is

(a) $y^2 = C(\ln\, x)^2 - 1$

(b) $y^2 = C \ln\, x - 1$

(c) $y = \sqrt{Cx^2 + 1}$

(d) $y = \sqrt{(\ln\, x)^2 + C}$

(e) None of the above.

9. If $y = y(x)$ is the solution of the initial-value problem $xy' + y = 4\cos 2x,\ y(\pi) = 0,$ then $\lim_{x\to 0} y(x) =$

   (a) ∞
   (b) 0
   (c) 2
   (d) 4
   (e) None of the above.

10. The general solution of $x^2y' - 2xy = 4x^4e^{2x}$ is

   (a) $y = -2x^2e^{-2x} + C$
   (b) $y = Cx^2 - 2x^2e^{2x}$
   (c) $y = -2xe^{-2x} + Cx^2$
   (d) $y = Cx^2e^{-2x}$
   (e) None of the above.

11. The solution of the initial-value problem $(y+1)y' = 2xy - y, \quad y(1) = 2$ is

   (a) $y + \ln|y| = x^2 - x$
   (b) $\ln|y+1| = x^2 - x + 2 + \ln 2$
   (c) $y = x^2 - x - 1$
   (d) $y - \ln|y| = x^2 - x + 1$
   (e) None of the above.

12. The general solution of $\dfrac{dy}{dx} = \dfrac{e^{x-y}}{1+e^x}$ is

   (a) $y = \ln[\ln(1+e^x)] + C$
   (b) $y = \ln[(1+e^x) + C]$
   (c) $y = \ln[\ln(1+e^x) + C]$
   (d) $y = \ln[C(1+e^x)]$
   (e) None of the above.

13. The general solution of $(x^2y + 2y)y' = 2xy^2 + 8x$ is:

   (a) $y^2 = C(x+2) - 4$
   (b) $y^2 = C(x+2)^2 - 4$
   (c) $y^2 = Cx^2 - 4$
   (d) $y = C\sqrt{x+2} + 4$
   (e) None of the above.

14. The general solution of $(1 + x^2 + y^2 + x^2y^2)\dfrac{dy}{dx} = 2x(1 + y^2)$ is

(a) $y^2 = y\ln(1 + x^2) + Cy - 1$

(b) $y^2 - 1 = y\tan^{-1} x + Cx$

(c) $y^2 = \dfrac{x^2y - y}{x} + Cy$

(d) $y = \ln(1 + x^2) + C$

(e) None of the above.

Given the linear operator $L[y] = y' + \dfrac{1}{x}\, y$.

15. Calculate $L(2x^2 + 3x)$.

(a) $2x + 3$

(b) $6x + 8$

(c) $6x + 6$

(d) $4x - 6$

(e) None of the above.

16. Find the set of all solutions of $L[y] = \dfrac{8\sin 4x}{x} + 2x^2$

(a) $y = \dfrac{x^5 - 4\cos 4x + C}{2x}$

(b) $y = 2x^4 + \dfrac{2\cos 4x + C}{x}$

(c) $y = \frac{1}{2}x^4 - 2\cos 4x + C$

(d) $y = \dfrac{x^4 + 4\cos 4x + C}{2x}$

(e) None of the above.