For problems 1 and 2, let let L be the linear operator defined by

$$L[y] = y'' - \frac{5}{x}\,y' + \frac{8}{x^2}\,y.$$

1. Calculate $L[x^2 - 2x^3]$.

   (a) $2x$

   (b) 4

   (c) $-4x$

   (d) $6x^2$

   (e) None of the above.

2. A fundamental set of solutions of $L[y] = 0$ is: (Hint: The equation has solutions of the form $y = x^r$.)

   (a) $\{x^{-2},\, x^{-4}\}$

   (b) $\{x^{-2},\, x^{6}\}$

   (c) $\{x^{2},\, x^{-4}\}$

   (d) $\{x^{2},\, x^{4}\}$

   (e) None of the above.

3. $y = x$ is a solution of $y'' - \frac{3}{x}\,y' + \frac{3}{x^2}\,y = 0$. Find a fundamental set of solutions of the equation. (Hint: See Problem Exercises 3.2, Problem 15.)

   (a) $\{x,\, x^{-1}\}$

   (b) $\{x,\, x^{-2}\}$

   (c) $\{x,\, x^{3}\}$

   (d) $\{x,\, x^{2}\}$

   (e) None of the above.

4. $y = x^2$ is a solution of $y'' - \frac{3}{x}\,y' + \frac{4}{x^2}\,y = 0$. Find a fundamental set of solutions of the equation. (Use the Hint in Problem 3.)

(a) $\{x^2,\ x^{-2}\}$

(b) $\{x^2,\ x^2 \ln x\}$

(c) $\{x^2,\ x \ln x\}$

(d) $\{x^2,\ x^3\}$

(e) None of the above.

5. $y'' + \frac{3}{x}y' - \frac{15}{x^2}y = 0$ has solutions of the form $y = x^r$. Give a fundamental set of solutions and find their Wronskian.

(a) $\{x^{-5},\ x^3\};\ W = 8x^{-3}$

(b) $\{x^{-3},\ x^5\};\ W = 8x$

(c) $\{x^{-5},\ x^{-3}\};\ W = 2x^{-9}$

(d) $\{e^{-5x},\ e^{3x}\};\ W = 2e^{-2x}$

(e) None of the above.

6. A solution basis for $y'' + 4y' - 12y = 0$ is:

(a) $\{e^{4x},\ e^{-3x}\}$

(b) $\{e^{-6x},\ e^{2x}\}$

(c) $\{e^{-4x},\ e^{3x}\}$

(d) $\{e^{6x},\ e^{-2x}\}$

(e) None of the above.

7. The general solution of $y'' + 3y' - 10y = 0$ is:

(a) $y = C_1e^{5x} + C_2e^{-2x}$

(b) $y = C_1e^{5x} + C_2e^{2x}$

(c) $y = C_1e^{-5x} + C_2e^{-2x}$

(d) $y = C_1e^{-5x} + C_2e^{2x}$

(e) None of the above.

8. The general solution of $y'' + 2y' + 10y = 0$ is:

(a) $C_1e^{-3x}\cos x + C_2e^{-3x}\sin x$

(b) $C_1e^{5x} + C_2e^{-2x}$

(c) $y = C_1e^{-x}\cos 3x + C_2e^{-x}\sin 3x$

(d) $C_1e^{5x} + C_2e^{-2x}$

(e) None of the above.

9. A solution basis for the differential equation $y'' + 14y' + 49y = 0$ is:

(a) $\{e^{7x},\ e^{-7x}\}$

(b) $\{e^{-7x},\ 1\}$

(c) $\{e^{-7x},\ xe^{-7x}\}$

(d) $\{e^{7x},\ xe^{7x}\}$

(e) None of the above.

10. The solution of the initial-value problem $y'' - 8y' + 16y = 0,\ y(0) = 1,\ y'(0) = 2$ is:

(a) $y = e^{4x} - 2xe^{4x}$

(b) $y = e^{-4x} - 6xe^{-4x}$

(c) $y = \frac{1}{2}e^{-4x} + \frac{3}{2}e^{4x}$

(d) $y = e^{4x} + 6xe^{4x}$

(e) None of the above.

11. A solution basis for the equation

$$y'' + 4y = 0$$

is:

(a) $\{e^{2x},\ e^{-2x}\}$

(b) $\{\cos 2x,\ \sin 2x\}$

(c) $\{0,\ e^{4x}\}$

(d) $\{\cos 4x,\ \sin 4x\}$

(e) None of the above.

12. A solution basis for the equation

$$y'' + 4y' = 0$$

is:

(a) $\{1,\ e^{4x}\}$

(b) $\{\sin 2x,\ e^{4x}\}$

(c) $\{x,\ e^{-4x}\}$

(d) $\{1,\ e^{-4x}\}$

(e) None of the above.

13. Which of the following equations does not have $y = e^{2x}$ as a solution?

(a) $y'' - 5y' + 6y = 0$

(b) $y'' + 2y' - 8y = 0$

(c) $y'' + 3y' - 10y = 0$

(d) $y'' - 2y' = 0$

(e) $y'' + y' - 2y = 0$

14. A fundamental set of solutions of $y'' + 4y' + 13y = 0$ is:

(a) $\{e^{-2x}\cos 3x,\ e^{-2x}\sin 3x\}$

(b) $\{e^{2x}\cos 3x,\ e^{2x}\sin 3x\}$

(c) $\{e^{3x}\cos 2x,\ e^{3x}\sin 2x\}$

(d) $\{e^{-3x}\cos 2x,\ e^{-3x}\sin 2x\}$

(e) None of the above.

15. $y = C_1e^{-x} + C_2e^{5x}$ is the general solution of a second order linear differential equation. The equation is:

(a) $y'' + 4y' - 5y = 0$

(b) $y'' - 6y' + 5y = 0$

(c) $y'' - 4y' - 5y = 0$

(d) $y'' + 6y' + 5y = 0$

(e) None of the above.

16. $y = 3e^{-2x} - xe^{-2x}$ is a solution of a second order linear differential equation with constant coefficients. The equation is:

(a) $y'' + 4y' + 4y = 0$

(b) $y'' + 4y' = 0$

(c) $y'' - 4y' + 4y = 0$

(d) $y'' - 4y = 0$

(e) None of the above.

17. $y = e^{-3x}\sin 4x$ is a solution of a second order linear differential equation with constant coefficients. The equation is:

(a) $y'' - 8y' + 25y = 0$

(b) $y'' + 6y' + 25y = 0$

(c) $y'' - 6y' + 16y = 0$

(d) $y'' - 6y' + 25y = 0$

(e) None of the above.

18. $y = 2e^{3x} + 4xe^{-4x}$ is a solution of a linear differential equation with constant coefficients. The equation is:

(a) $y'' + y' - 12y = 0$

(b) $y'' + 7y' - 12y = 0$

(c) $y'' + y' + 12y = 0$

(d) $y'' - 7y' + 12y = 0$

(e) None of the above.