1. The differential equation that has $y = C_1 + C_2 x^3 - 2x$ as its general solution is:

   (a) $xy'' - 2y' = 4$.

   (b) $x^2 y'' - 2y = x$.

   (c) $y'' - 2xy' - 4 = 0$.

   (d) $y'' - 2y = 2x$.

   (e) none of the above.

2. The differential equation which has $y^3 = Cx^4 - 3x$ as its general solution is:

   (a) $y' = \dfrac{y^3 + 3x}{3xy^2}$

   (b) $y' = \dfrac{4y^3 + 9x}{3xy^2}$

   (c) $y' = \dfrac{4y^3 + 12x}{3xy^2}$

   (d) $y' = \dfrac{4y^3 + 12}{3xy^2}$

   (e) None of the above.

3. The general solution of $xy' = 6x^3 e^{2x} + 2y$ is:

   (a) $y = 3x^2 e^{2x} + Cx^2$.

   (b) $y = 3x^2 e^{2x} + C$.

   (c) $y = 3e^{2x} + Cx^2$.

   (d) $y = 3x^2 e^{2x} - xe^{2x} + Cx^2$.

   (e) none of the above.

4. The general solution of $(x^2 y + 5y)y' = 2xy^2 + 8x$ is:

   (a) $y^2 + 4 = (x^2 + 5) + C$

   (b) $y^2 + 4 = (x^2 + 5)^2 + C$

   (c) $y^2 = C(x^2 + 5) - 4$

   (d) $y^2 = C(x^2 + 5)^2 - 4$

   (e) None of the above

5. The general solution of $y' + xy = xy^3$ is

   (a) $y = \dfrac{1}{1 + Ce^{x^2}}$

   (b) $y^2 = \dfrac{1}{1 + Ce^{x^2}}$

(c) $y = \sqrt{1 + Ce^{x^2}}$

(d) $y^2 = \dfrac{1}{1 + Ce^{-x^2}}$

(e) none of the above.

6. The general solution of $y' = \dfrac{x^2 e^{y/x} + y^2}{xy}$. is

    (a) $ye^{y/x} + xe^{y/x} = Cx - x\ln x$

    (b) $ye^{-y/x} + xe^{-y/x} = x - x\ln x + C.$

    (c) $ye^{-y/x} + xe^{-y/x} = Cx - x\ln x$

    (d) $ye^{-y/x} + xe^{-y/x} = Cx + x\ln x$

    (e) none of the above.

7. The general solution of $xy^2\, y' = x^3 + y^3$ is:

    (a) $y^2 = x^2 \ln Cx^2$

    (b) $y^3 = x\ln Cx^3$

    (c) $y^2 = x^2 \ln x^3 + Cx^2$

    (d) $y^3 = x^3 \ln Cx^3$

    (e) None of the above.

8. The family of orthogonal trajectories of $y^3 = Cx^4 - 2$ is:

    (a) $3x^2 + 4y^2 - \dfrac{8}{y} = C$

    (b) $x^2 + 2y^2 + 8\ln y = C$

    (c) $3x^2 + 4y^2 - \dfrac{16}{y} = C$

    (d) $4y^2 + \dfrac{16}{y} = 3x^2 + C$

    (e) none of the above.

9. A sample of 100 grams of radioactive material was present initially and after 3 hours the sample lost 20% of its mass. How much material will remain after 9 hours?

    (a) $A(9) = 100\left(\dfrac{4}{5}\right)^3$

    (b) $A(9) = 100\left(\dfrac{3}{5}\right)^2$

    (c) $A(9) = 100\left(\dfrac{4}{5}\right)^9$

    (d) $A(9) = 100\left(\dfrac{3}{5}\right)^{-3}$

(e) None of the above.

10. A colony of penguins obeys the population growth law. Initially the colony had 75 penguins. Three years later the colony had 100 penguins. What is the size of the colony after 12 years?

(a) $P(12) = 100 \left(\dfrac{3}{4}\right)^4$

(b) $P(12) = 75 \left(\dfrac{3}{2}\right)^3$

(c) $A(12) = 100 \left(\dfrac{3}{4}\right)^3$

(d) $P(12) = 75 \left(\dfrac{4}{3}\right)^4$

(e) None of the above.

11. A disease is spreading through a small cruise ship with 200 passengers. Let $P(t)$ be the number of people who have disease at time t. The disease is spreading at a rate proportional to the product of the time elapsed and the number of people who are not sick. Suppose that 20 people have the disease initially. The mathematical model for the spread of the disease is:

(a) $\dfrac{dP}{dt} = k(200 - P), \ P(0) = 20.$

(b) $\dfrac{dP}{dt} = kt(200 - P), \ P(0) = 20.$

(c) $\dfrac{dP}{dt} = ktP, \ P(0) = 20.$

(d) $\dfrac{dP}{dt} = kP(200 - P), \ P(0) = 20.$

(e) None of the above.

12. Refer to Problem 11. Suppose that 50 people are sick after 4 days. Then the number of people that are sick at any time t is given by:

(a) $P(t) = 200 - 180 \left(\dfrac{5}{6}\right)^{t/4}$

(b) $P(t) = 20 \left(\dfrac{5}{2}\right)^{t^2/16}$

(c) $P(t) = 200 - 180 \left(\dfrac{5}{6}\right)^{t^2/16}$

(d) $P(t) = \dfrac{200}{2 + 18(5/6)^{t/4}}$

(e) None of the above.

13. $y'' - \dfrac{2}{x} y' - \dfrac{10}{x^2} y = 0$ has solutions of the form $y = x^r$. The general solution of the equation is:

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(a) $y = C_1 x^2 + C_2 x^{-5}$

(b) $y = C_1 x^9 + C_2 x^{-1}$

(c) $y = C_1 x^{-2} + C_2 x^5$

(d) $y = C_1 x^2 + C_2 x^5$

(e) None of the above.

14. Find the solution of the initial-value problem

$$x^2 \, y'' - 6 \, y = 0, \quad y(1) = 6, \ y'(1) = -2.$$

Hint: The equation has solutions of the form $y = x^r$.

(a) $y = 2x^3 + 4x^{-2}$

(b) $y = 4x^6 - 2x^{-1}$

(c) $y = 4x^3 + 2x^{-2}$

(d) $y = 2x^{-6} + 4x$

(e) None of the above.

15. The general solution of $y'' - 8y' + 20y = 0$ is:

(a) $y = C_1 e^{2x} \cos 4x + C_2 e^{2x} \sin 4x$

(b) $y = C_1 e^{10x} + C_2 e^{-2x}$

(c) $y = C_1 e^{5x} + C_2 e^{4x}$

(d) $y = C_1 e^{4x} \cos 2x + C_2 e^{4x} \sin 2x$

(e) None of the above.

16. A fundamental set of solutions of $y'' - 4y' - 12y = 0$ is:

(a) $\{e^{4x}, \ e^{-3x}\}$

(b) $\{e^{6x}, \ e^{-2x}\}$

(c) $\{e^{2x}, \ e^{-6x}\}$

(d) $\{e^{-4x}, \ e^{3x}\}$

(e) None of the above.

17. The general solution of $y'' + 10y' + 25y = 0$ is:

(a) $y = C_1 e^{5x} + C_2 x e^{5x}$

(b) $y = C_1 e^{-5x} + C_2 e^{5x}$

(c) $y = C_1 e^{5x} + C_2 x e^{-5x}$

(d) $y = C_1 e^{-5x} + C_2 x e^{-5x}$

(e) None of the above.

18. A solution basis for $y'' + 8y' + 16y = 0$ is:

(a) $\{e^{4x},\ xe^{4x}\}$

(b) $\{e^{4x},\ e^{-4x}\}$

(c) $\{e^{-4x},\ xe^{-4x}\}$

(d) $\{e^{4x},\ xe^{-4x}\}$

(e) None of the above.

19. The second order linear differential equation that has $y = 2e^{-2x} - e^{4x}$ as a solution is:

(a) $y'' - 2y' - 8y = 0$

(b) $y'' - 6y' + 8y = 0$

(c) $y'' + 2y' - 8y = 0$

(d) $y'' - 6y' - 8y = 0$

(e) None of the above.

20. The second order linear differential equation that has $y = 4xe^{-3x}$ as a solution is:

(a) $y'' - 6y' + 9y = 0$

(b) $y'' + 3y' = 0$

(c) $y'' + 6y' + 9y = 0$

(d) $y'' - 9y = 0$

(e) None of the above.

21. The second order linear differential equation that has $y = 3e^{-2x}\cos 2x$ as a solution is:

(a) $y'' - 4y' + 12y = 0$

(b) $y'' + 4y' + 8y = 0$

(c) $y'' - 8y' + 8y = 0$

(d) $y'' - 4y' + 8y = 0$

(e) None of the above.

22. The second order linear differential equation that has $y = 3e^{2x} - 2xe^{-3x}$ as a solution is:

(a) $y'' + y' - 6y = 0$

(b) $y'' - y' - 6y = 0$

(c) $y'' - 5y' + 6y = 0$

(d) $y'' + 5y' + 6y = 0$

(e) None of the above.