# MATH 3321
# Sample Questions for Exam 2

## Linear Nonhomogeneous Differential Equations

1. Find the general solution of $y'' - \dfrac{4}{x}\,y' + \dfrac{6}{x^2}\,y = \dfrac{4}{x^2}$.

   **Answer** $y = C_1\,x^2 + C_2\,x^3 + \frac{2}{3}$.

2. Find the general solution of $y'' + 4\,y = 2\,\tan\,2x$.

   **Answer** $y = C_1\,\cos\,2x + C_2\,\sin\,2x - \frac{1}{2}\,\sin\,2x\,\ln\,|\sec\,2x + \tan\,2x|$.

3. Find the general solution of $y'' - 6\,y' + 9\,y = 4\,e^{3x} + \dfrac{e^{3x}}{x}$.

   **Answer** $y = C_1\,e^{3x} + C_2\,x\,e^{3x} + 2x^2e^{3x} + x\,e^{3x}\,\ln\,x$.

4. Find the general solution of $y'' + 9\,y = 4\,\cos\,2x$.

   **Answer** $y = C_1\,\cos\,3x + C_2\,\sin\,3x + \frac{4}{5}\,\cos\,2x$.

5. Find the general solution of $y'' + 4\,y = 2\,\sin\,2x$.

   **Answer** $y = C_1\,\cos\,2x + C_2\,\sin\,2x - \frac{1}{2}\,x\,\cos\,2x$.

6. Find the general solution of $y'' - 6\,y' + 8\,y = 2\,e^{4x} + 6$.

   **Answer** $y = C_1\,e^{4x} + C_2\,e^{2x} + 2x\,e^{4x} + \frac{3}{4}$.

7. A particular solution of the nonhomogeneous differential equation

   $$y'' - 2\,y' - 15\,y = 2\,\cos\,3x + 5\,e^{5x} + 2$$

   will have the form:

   **Answer** $z = A\,\cos\,3x + B\,\sin\,3x + C\,x\,e^{5x} + D$.

8. A particular solution of the nonhomogeneous differential equation

   $$y'' - 8\,y' + 16\,y = e^{2x}\,\sin\,4x + 2\,e^{4x} + 5x$$

   will have the form:

   **Answer** $A\,e^{2x}\,\cos\,4x + B\,e^{2x}\,\sin\,4x + C\,x^2\,e^{4x} + Dx + E$.

**Laplace Transformations**

1. Find the Laplace transform of $f(x) = 2\,e^{-3x} + \cos\,2x + 5x$.

   **Answer** $F(s) = \dfrac{2}{s+3} + \dfrac{s}{s^2+4} + \dfrac{5}{s^2}.$

2. Find the Laplace transform of $f(x) = 3x\,e^{\beta x} + e^{x}\sin\,3x$.

   **Answer** $F(s) = \dfrac{3}{(s-\beta)^2} + \dfrac{3}{s^2-2s+10}.$

3. If $F(s) = \dfrac{2}{s^2} + \dfrac{s-3}{s^2+4}$, then $\mathcal{L}^{-1}[F(s)]$ is:

   **Answer** $f(x) = 2x + \cos\,2x - \frac{3}{2}\sin\,2x.$

4. If $F(s) = \dfrac{1}{(s-3)^2} + \dfrac{s+2}{s^2-4s+13}$, then $\mathcal{L}^{-1}[F(s)]$ is:

   **Answer** $f(x) = x\,e^{3x} + e^{2x}\cos\,3x + \frac{4}{3}\,e^{2x}\sin\,3x.$

5. Find the Laplace transform of the solution of the initial-value problem

$$y' + 2y = 3\cos\,2x;\;\; y(0) = 3.$$

   **Answer** $Y(s) = \dfrac{9}{4(s+2)} + \dfrac{3s+6}{4(s^2+4)} + \dfrac{3}{s+2}.$

6. Find the Laplace transform of the solution of the initial-value problem

$$y'' - 5\,y' + 6\,y = 4\sin\,3x;\;\; y(0) = 0,\;\; y'(0) = 2.$$

   **Answer** $F(s) = \dfrac{12}{(s^2+9)(s^2-5s+6)} + \dfrac{2}{s^2-5s+6}.$

7. Find the Laplace transform of the solution of the initial-value problem

$$y'' + 25\,y = 2\,e^{-3x};\;\; y(0) = 2,\;\; y'(0) = 0.$$

   **Answer** $F(s) = \dfrac{1}{17(s+3)} - \dfrac{s+5}{17(s^2+25)} + \dfrac{2s}{s^2+25}.$

8. Use the Laplace transform method to find the solution of the initial-value problem

$$y' - 3\,y = 2\,e^{2x};\;\; y(0) = 1.$$

   **Answer** $y = 3e^{3x} - 2e^{2x}.$

9. Find the value(s) of $\gamma$ such that the solution of the initial-value problem

$$y'' - 4\,y = \sin\,x;\; y(0) = \gamma,\; y'(0) = 0$$

is bounded.

**Answer** $\gamma = -\frac{1}{10}$.

10. Find the value of $\delta$ such that the solution of the initial-value problem

$$y' - 3\,y = 2\,e^{-2x};\; y(0) = \delta$$

has limit $0$ as $x \to \infty$.

**Answer** $\delta = -\frac{2}{5}$.

11. If $F(s) = \dfrac{4}{s^2} + \dfrac{3s+2}{s^2+9}$, then $\mathcal{L}^{-1}[F(s)]$ is:

**Answer** $f(x) = 4x + 3\,\cos\,3x + \frac{2}{3}\,\sin\,3x$.

12. If $F(s) = \dfrac{2s+3}{(s-3)(s^2+4)}$, then $\mathcal{L}^{-1}[F(s)]$ is:

**Answer** $f(x) = \frac{9}{13}\,e^{3x} - \frac{1}{26}\,\sin\,2x - \frac{9}{13}\,\cos\,2x$.

13. If

$$f(x) = \begin{cases} x^2 & 0 \le x < 3 \\ 2 & x \ge 3 \end{cases}$$

then $\mathcal{L}[f(x)] =$

**Answer** $F(s) = \dfrac{2}{s^3} - 2e^{-3s}\dfrac{1}{s^3} - 6\,e^{-3s}\dfrac{1}{s^2} - 7e^{-3s}\dfrac{1}{s}$.

14. If

$$f(x) = \begin{cases} -2 & 0 \le x < 2 \\ x & 2 \le x < 5 \\ 3 & x \ge 5 \end{cases}$$

then $\mathcal{L}[f(x)] =$

**Answer** $F(s) = -\dfrac{2}{s} - e^{-2s}\dfrac{1}{s^2} + 4\,e^{-2s}\dfrac{1}{s} - e^{-5s}\dfrac{1}{s^2} - 2\,e^{-5s}\dfrac{1}{s}$.

15. If $F(s) = \dfrac{2}{s} + e^{-3s}\dfrac{1}{s^2} + 4\,e^{-3s}\dfrac{1}{s}$, then $\mathcal{L}^{-1}[F(s)]$ is:

**Answer** $f(x) = \begin{cases} 2 & 0 \le x < 3 \\ 3 + x & x \ge 3 \end{cases}$

16. If $F(s) = \dfrac{3s + (s-2)e^{-\pi s}}{s^2+9}$, then $\mathcal{L}^{-1}[F(s)] = f(x)$ is:

**Answer** $f(x) = \begin{cases} 3\cos 3x & 0 \le x < \pi \\ 2\cos 3x + \frac{2}{3}\sin 3x & x \ge \pi \end{cases}$

**Systems of Linear Algebraic Equations**

1. Find the solution set of the system of linear equations

$$\begin{aligned} 4x - y + 2z &= 3 \\ -4x + y - 3z &= -10 \\ 8x - 2y + 9z &= -1 \end{aligned}$$

**Answer** No solution.

2. Find the solution set of the system of linear equations

$$\begin{aligned} 2x - 5y - 3z &= 7 \\ -4x + 10y + 2z &= 6 \\ 6x - 15y - z &= -19 \end{aligned}$$

**Answer** $x = \frac{5}{2}a - 4,\ y = a,\ z = -5,\ a$ any real number.

3. Find the solution set of the system of linear equations

$$\begin{aligned} 5x - 3y + 2z &= 13 \\ 2x - y - 3z &= 1 \\ 4x - 2y + 4z &= 12 \end{aligned}$$

**Answer** $x = 1,\ y = -2,\ z = 1.$

4. Find the solution set of the system of linear equations

$$\begin{aligned} x - y + z &= 1 \\ 4x + y + z &= 5 \\ 2x + 3y - z &= 2 \end{aligned}$$

**Answer** No solution.

5. Find the solution set of the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - x_3 - x_4 &= 0 \\ x_1 + 2x_2 + x_4 &= 4 \\ -x_1 - 2x_2 + 2x_3 + 4x_4 &= 5 \end{aligned}$$

**Answer** $x_1 = 3 - 2a,\ x_2 = a,\ x_3 = 2,\ x_4 = 1,\ a$ any real number.

6. Find the solution set of the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + 4x_4 = 2 \\ 2x_1 + 5x_2 - 2x_3 + x_4 = 1 \\ 5x_1 + 12x_2 - 7x_3 + 6x_4 = 7 \end{aligned}$$

**Answer** No solution.

7. Find the solution set of the system of linear equations

$$\begin{aligned} x_1 + 2x_2 - 3x_3 - 4x_4 = 2 \\ 2x_1 + 4x_2 - 5x_3 - 7x_4 = 7 \\ -3x_1 - 6x_2 + 11x_3 + 14x_4 = 0 \end{aligned}$$

**Answer** $x_1 = 11 - 2a + b,\ x_2 = a,\ x_3 = 3 - b,\ x_4 = b,\ a,\ b$ any real numbers.

8. Find the solution set of the system of linear equations

$$\begin{aligned} x + 2y - 3z = 0 \\ 2x + 5y + 2z = 0 \\ 3x - y - 4z = 0 \end{aligned}$$

**Answer** $x = y = z = 0.$

9. Find the solution set of the system of linear equations

$$\begin{aligned} 3x_1 + x_2 - 5x_3 - x_4 = 0 \\ 2x_1 + x_2 - 3x_3 - 2x_4 = 0 \\ x_1 + x_2 - x_3 - 3x_4 = 0 \end{aligned}$$

**Answer** $x_1 = 2a - b,\ x_2 = -a + 4b,\ x_3 = a,\ x_4 = b,\ a,\ b$ any real numbers.

10. For what values of $a$ and $b$ does the system

$$\begin{aligned} -x - 2z &= a \\ 2x + y + x &= 0 \\ x + y - z &= b \end{aligned}$$

have a solution?

**Answer** $b = a,\ a$ any real number.

11. For what values of $a$ does the system

$$\begin{aligned} x + ay - 2z &= 0 \\ 2x - y - z &= 0 \\ -x - y + z &= 0 \end{aligned}$$

have nontrivial solutions?

**Answer** $a = 4.$

12. Find the value(s) of $k$, if any, so that the system of equations

$$\begin{aligned} x - 2y &= 1 \\ x - y + kz &= -2 \\ ky + 4z &= 6 \end{aligned} \; .$$

has a unique solution.

**Answer** $k \neq \pm 2$.

13. Find the value(s) of $k$, if any, so that the system of equations

$$\begin{aligned} x - 2y &= 1 \\ x - y + kz &= -2 \\ ky + 4z &= 6 \end{aligned} \; .$$

has infinitely many solutions.

**Answer** $k = -2$.