MATH 3321
Sample Questions for Exam 3

1. Find x and y so that $\begin{pmatrix} 2x & 4 \\ -3 & 5x \end{pmatrix} + \begin{pmatrix} 3y & -2 \\ -2 & -y \end{pmatrix} = \begin{pmatrix} -5 & 2 \\ -5 & 12 \end{pmatrix}$.

2. Let $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 1 \\ 2 & 5 \end{pmatrix}$, $C = \begin{pmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{pmatrix}$.

   Perform the indicated operations, if possible: (a) AC (b) AB (c) $B + AC$ (d) CBA

3. Show that the matrix of coefficients is non-singular and use Cramer's rule to find the unique solution of the system:

$$\begin{aligned} x_1 - x_2 + x_3 &= 1 \\ 2x_2 - x_3 &= 1 \\ 2x_1 + 3x_2 &= 1 \end{aligned}$$

4. Write the system in the vector-matrix form $A\mathbf{x} = \mathbf{b}$ and solve by finding A^{-1}.

$$\begin{aligned} 2x_1 + x_2 &= 2 \\ 4x_1 + 3x_2 &= -4 \end{aligned}$$

5. Write the system in the vector-matrix form $A\mathbf{x} = \mathbf{b}$ and solve by finding A^{-1}.

$$\begin{aligned} x_1 + x_2 &= 2 \\ 2x_1 + 3x_2 - x_3 &= 0 \\ x_1 + 2x_3 &= 4 \end{aligned}$$

6. Given the set of vectors

$$\{\mathbf{u} = (1, -2, 1), \ \mathbf{v} = (2, 1, -1), \ \mathbf{w} = (7, -4, 1)\}$$

   Is the set linearly dependent or linearly independent? If it is linearly dependent, express one of the vectors as a linear combination of the other two.

7. Given the set of vectors

$$\{\mathbf{v}_1 = (2, 0, -1), \ \mathbf{v}_2 = (-3, 1, 2), \ \mathbf{v}_3 = (8, -2, -5), \ \mathbf{v}_4 = (-9, 1, 5)\}$$

   Is the set linearly dependent or linearly independent? If it is linearly dependent, how many independent vectors are there in the set?

8. For what values of a are the vectors

$$\{\mathbf{v}_1 = (\,a,\ 1,\ -1\,),\ \mathbf{v}_2 = (\,-1,\ 2a,\ 3\,),\ \mathbf{v}_3 = (\,-2,\ a,\ 2\,),\ \mathbf{v}_4 = (\,3a,\ -2,\ a\,)\}$$

linearly dependent?

9. For what values of a are the vectors

$$\{\mathbf{u} = (\,a,\ 1,\ -1\,),\ \mathbf{v} = (\,-1,\ 2a,\ 3\,),\ \mathbf{w} = (\,-2,\ a,\ 2\,)\}$$

linearly dependent?

10. Given the set of functions $\{f_1(x) = 1 + x,\ f_2(x) = 1 - x,\ f_3(x) = x^2 - 1\}$. Calculate the Wronskian of the functions. Is the set linearly dependent or linearly independent?

11. Given the set of functions $\{f_1(x) = 1 + 2x,\ f_2(x) = 1 - x,\ f_3(x) = 3x - 1\}$. Calculate the Wronskian of the functions. Is the set linearly dependent or linearly independent? If it is linearly dependent, express one of the functions as a linear combination of the other two.

12. The eigenvalues and eigenvectors of $\begin{pmatrix} 6 & -2 & 0 \\ 4 & 0 & 0 \\ 4 & 1 & -1 \end{pmatrix}$ are: (Hint: 4 is an eigenvalue.)

13. The eigenvalues and eigenvectors of $\begin{pmatrix} 3 & -2 & 1 \\ 2 & -2 & 2 \\ 2 & -3 & 3 \end{pmatrix}$ are: (Hint: 1 is an eigenvalue.)

14. The eigenvalues and eigenvectors of $\begin{pmatrix} 1 & 1 & 0 \\ -2 & 4 & 0 \\ -2 & 2 & 2 \end{pmatrix}$ are: (Hint: 2 is an eigenvalue.)

15. The eigenvalues and eigenvectors of $\begin{pmatrix} 1 & -2 \\ 5 & 3 \end{pmatrix}$ are:

16. The general solution of $y''' - 4\,y'' + y' + 6\,y = 0$ is: (Hint: $r = 2$ is a root of the characteristic equation)

17. The general solution of $y''' + y'' - 8\,y' - 12\,y = 0$ is: (Hint: $r = 3$ is a root of the characteristic equation)

18. The general solution of $y^{(4)} + 2\,y''' + 4\,y'' - 2\,y' - 5\,y = 0$ is: (Hint: $r = -1 + 2i$ is a root of the characteristic equation)

19. The homogeneous equation with constant coefficients of least order that has

$$y = 2e^{3x} + 3\sin\,2x + 2x$$

as a solution is:

20. A particular solution of $y^{(4)} - 16\,y = 2e^{-2x} + 3e^{4x} + \cos\,2x + 5$ will have the form:

21. Given the differential equation $y''' - 4y'' - 3y' + 18y = 0$.

   (a) Write the equation in the vector-matrix form $\mathbf{x}' = A\mathbf{x}$.

   (b) Find three linear independent solution vectors of the system in (a), given that -2 is a root of the characteristic polynomial.

22. Find the solution of the initial-value problem $\mathbf{x}' = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}\mathbf{x}$; $\mathbf{x}(0) = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$. HINT: 2 is a root of the characteristic polynomial.

23. Find a fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} -1 & -4 \\ 1 & -1 \end{pmatrix}\mathbf{x}$.

24. Find the general solution of $\mathbf{x}' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}\mathbf{x}$. HINT: 2 is a root of the characteristic polynomial.

25. Find a fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}\mathbf{x}$. HINT: 10 is a root of the characteristic polynomial.