MATH 1431 **FINAL EXAM REVIEW**

Problem 1.

a. In each of the following, determine whether the given limit exists. If the limit exists, give its value:

(i) $\displaystyle\lim_{x\to 0}\sqrt{x^2+3+\cos x}$ (ii) $\displaystyle\lim_{x\to 0}\frac{\sin 2x\,\cos x}{x}$

(iii) $\displaystyle\lim_{x\to 2^+}\frac{x^2-9}{x^2-3x+2}$ (iv) $\displaystyle\lim_{x\to 1} f(x)$

where $f(x)=\begin{cases} 2x+1, & x\le 1\\ x^2-2x, & x>1.\end{cases}$

b. Let $f(x)=\dfrac{x^2-4}{x^3-5x^2+6x}$.

(i) Determine the intervals on which f is continuous.

(ii) Determine the vertical asymptotes of the graph of f, if any.

(iii) Determine the horizontal asymptotes of the graph of f, if any.

Problem 2.

a. Let $f(x)=\begin{cases} 3x^2-1, & x<1\\ A, & x=1\\ x^3+2Bx, & x>1.\end{cases}$ Find A and B so that f will be continuous at $x=1$.

b. Let $g(x)=\begin{cases} x+2, & x<-1\\ x^2, & -1\le x\le 1\\ 1/(x-1), & x>1.\end{cases}$

(i) Sketch the graph of g.

(ii) What is $\displaystyle\lim_{x\to -1} g(x)$? What is $\displaystyle\lim_{x\to 1} g(x)$?

(iii) At what numbers x, if any, does g fail to be continuous?

(iv) At what numbers x, if any, does g fail to be differentiable?

Problem 3.

a. Use the definition of the derivative to calculate $f'(3)$ where $f(x)=1+3x-2x^2$.

b. Calculate the derivative of each of the following functions:

(i) $f(x)=3x^4-\dfrac{3}{x^2}+\sqrt{x}+\sqrt{2}$ (ii) $g(x)=\dfrac{x^2+\cos x}{\sin 2x}$

(iii) $y=x^3\tan(x^2+1)$ (iv) $h(x)=\left(x^{3/2}+\sec 3x\right)^4$

Problem 4.

a. Determine $\dfrac{dy}{dx}$ if y is defined implicitly by the equation: $x^3 - 3x^2y^2 + x^2 \sin y = y^3$.

b. Determine equations for the tangent and normal lines to the graph of $x^2 + y^2 = 5y$ at the point $(-2, 1)$.

Problem 5.

a. A helium balloon is rising at the rate of 10 feet per second. A 25 foot lamppost is 20 feet from the point directly below the rising balloon. At what rate is the shadow of the balloon moving along the ground at the instant it is 15 feet above the ground?

b. A conical water tank (vertex down) is initially full of water. The tank is 8 feet deep and the radius of the top is 2 feet. If the water is being drained from the bottom of the tank at the rate of $\frac{1}{2}$ ft^3 /min, how fast is the water level falling at the instant the water is 5 feet deep?

Problem 6.

a. Find the critical numbers of the function $f(x) = \frac{1}{3}x(x-3)^{2/3}$, and for each critical number c determine whether f has a local maximum, a local minimum, or neither a maximum nor a minimum at c. Does the graph of f have a vertical tangent or a cusp?

b. Determine the absolute maximum and the absolute minimum values of $g(x) = x^4 - 8x^2 + 3$ on $[-3, 1]$.

Problem 7. Let $f(x) = (x-1)^3(x+3)$.

a. Determine the critical numbers of f, find the intervals on which f is increasing and the intervals on which f is decreasing.

b. Determine the intervals on which the graph of f is concave up, the intervals on which the graph is concave down, and find the points of inflection.

c. Sketch the graph of f.

Problem 8. Let $g(x) = x + \sin 2x, \quad x \in [0, \pi]$.

a. Determine the critical numbers of g, find the intervals on which g is increasing and the intervals on which f is decreasing.

b. Determine the intervals on which the graph of g is concave up, the intervals on which the graph is concave down, and find the points of inflection.

c. Sketch the graph of g.

Problem 9.

a. A farmer with 1000 feet of fencing wants to enclose a rectangular area and then divide it into three pens with fencing parallel to one of the sides.

  (i) Let $A = xy$ be the area of the rectangle. Express A either as a function of x or as a function of y, and give the domain of the function.

  (ii) Determine the dimensions x, y such that the rectangle encloses maximum area.

  (iii) What is the maximum area?

b. A closed rectangular container with a square base of side length x is to have a volume of 2000 cubic inches. The material for the top and bottom of the container costs 20 cents per square inch, and the material for the sides costs 10 cents per square inch.

  (i) Express the cost C as a function of x and give the domain of the function.

  (ii) Determine x such that C is a minimum.

  (iii) What is the minimum cost?

Problem 10.

a. Evaluate the definite integral: $\displaystyle\int_{-1}^{1} (x+1)(x^2+2x)^3\,dx.$

b. Let $G(x) = \displaystyle\int_{1}^{x^3} 4t\sec^2\left(2t^2+1\right)\,dt$. Calculate $G(1)$ and $G'(x)$.

c. Show that $H(x) = \frac{2}{3}(x-2)\sqrt{x+1}$ is an antiderivative of $h(x) = \dfrac{x}{\sqrt{x+1}}$. Determine the area of the region bounded by the graph of h and the x-axis on $[0,3]$. Note: $h(x) \geq 0$ on $[0,3]$.

Problem 11. Evaluate each of the following indefinite integrals:

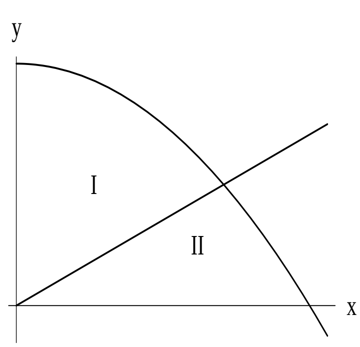
a. $\displaystyle\int \left(3x^4 - \frac{3}{x^2} + \sqrt{x} + \sqrt{2}\right) dx$

b. $\displaystyle\int \frac{1}{\sqrt{1+\cos x}}\sin x\,dx$

c. $\displaystyle\int \sec x\,[\sec x - \tan x]\;dx$

d. $\displaystyle\int (x+1)\left(2x^2+4x+1\right)^3\,dx$

Problem 12. The graphs of the functions $f(x) = 8 - x^2$ and $g(x) = 2x$ are shown in the figure.



a. Determine the area of region I.

b. Region II is revolved about the x-axis. Set up the definite integral (or integrals) for the volume of the solid that is generated by integrating (i) with respect to x; and (ii) with respect to y.

c. Region I is revolved about the y-axis.

(i) Set up the definite integral (or integrals) for the volume, integrating with respect to x.

(ii) Set up the definite integral (or integrals) for the volume, integrating with respect to y.

(iii) Calculate the volume of the solid.

Problem 13. The base of a solid is the circular region with boundary $x^2 + y^2 = 9$.

a. Find the volume of the solid given that cross-sections perpendicular to the x-axis are squares.

b. Find the volume of the solid given that cross-sections perpendicular to the x-axis are semi-circles.

Problem 14. Let $f(x) = 4x - x^2$ on $[0, 4]$, and let Ω be the region bounded by the graph of f and the x-axis.

a. Find the centroid of Ω.

b. Use Pappus's theorem to find the volume of the solid generated by revolving Ω about:

(i) the x-axis,

(ii) the y-axis.