MATH 3321
Quiz #8

1. The eigenvalues and eigenvectors of $\begin{pmatrix} 0 & 2 \\ 3 & -1 \end{pmatrix}$ are:

   Answer $2, \begin{pmatrix} 1 \\ 1 \end{pmatrix}; \quad -3, \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

2. The eigenvalues and independent eigenvectors of $\begin{pmatrix} 9 & -5 & 0 \\ 10 & -6 & 0 \\ 10 & -9 & 3 \end{pmatrix}$ are: (Hint: 4 is an eigenvalue.)

   Answer $4, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}; \ 3, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \ -1, \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$

3. The eigenvalues and independent eigenvectors of $\begin{pmatrix} 4 & -2 & 1 \\ 2 & -1 & 2 \\ 2 & -3 & 4 \end{pmatrix}$ are: (Hint: 2 is an eigenvalue.)

   Answer $2, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}; \ 3, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

4. The eigenvalues and independent eigenvectors of $\begin{pmatrix} -5 & 3 & 0 \\ -6 & 4 & 0 \\ -6 & 6 & -2 \end{pmatrix}$ are: (Hint: 1 is an eigenvalue.)

   Answer $1, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}; \ -2, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

   parameters: $\alpha,\ \beta = \pm 1,\ \pm 2,\ \pm 3,\ \pm 4;\ \alpha \neq \beta,\ \alpha \neq 2\beta.$

5. The eigenvalues and independent eigenvectors of $\begin{pmatrix} 6 & -3 & -1 \\ 8 & -5 & -1 \\ 10 & -8 & 1 \end{pmatrix}$ are: Hint: -2 is an eigenvalue.)

   Answer $-2, \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}; \ 2+i, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + i \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}; \ 2-i, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - i \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

6. The general solution of $y''' - 3\,y'' - 4\,y' + 12\,y = 0$ is: (Hint: 2 is a root of the characteristic polynomial.)

   Answer $y = C_1 e^{2x} + C_2 e^{3x} + C_3 e^{-2x}$

7. The general solution of $y''' + y'' - 8\,y' - 12\,y = 0$ is: (Hint: 3 is a root of the characteristic polynomial.)

   Answer $y = C_1 e^{-2x} + C_2 x e^{-2x} + C_3 e^{3x}$

8. The general solution of $y^{(4)} - 4\,y''' + 12\,y'' + 4\,y' - 13\,y = 0$ is: (Hint: $2 + 3i$ is a root of the characteristic polynomial.)

   Answer $y = C_1 e^{x} + C_2 e^{-x} + C_3 e^{2x} \cos\, 3x + C_4 e^{2x} \sin\, 3x$

9. The homogeneous equation with constant coefficients of least order that has

$$y = 2e^{\alpha x} + 3 \sin\, \beta x + 2x$$

   as a solution is:

   Answer $y^{(5)} - 3\,y^{(4)} + 4\,y''' - 12\,y'' = 0.$

10. A particular solution of $y^{(4)} - 16\,y = 2e^{-2x} + 3e^{3x} + \cos\, 2x + 5$ will have the form:

    Answer $y = Axe^{-2x} + Be^{3x} + Cx \cos\, 2x + Dx \sin\, 2x + E.$

11. A solution of the initial-value problem $\mathbf{x}' = \begin{pmatrix} 0 & 4 \\ -2 & 6 \end{pmatrix} \mathbf{x},\ \mathbf{x}(0) = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ is:

    Answer $\mathbf{x}(t) = -5e^{4t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + 3e^{2t} \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$

12. The general solution of $\mathbf{x}' = \begin{pmatrix} 9 & -5 & 0 \\ 10 & -6 & 0 \\ 10 & -9 & 3 \end{pmatrix} \mathbf{x}$ is:

    Answer $\mathbf{x}(t) = C_1\, e^{4t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + C_3\, e^{-t} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}.$

13. The general solution of $\mathbf{x}' = \begin{pmatrix} -7 & 4 & 1 \\ -10 & 6 & 2 \\ -10 & 3 & 5 \end{pmatrix} \mathbf{x}$ is:

Answer $\mathbf{x}(t) = C_1\, e^{-2t} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} + C_2\, e^{3t} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + C_3 \left[e^{3t} \begin{pmatrix} -3/10 \\ -1 \\ 0 \end{pmatrix} + te^{3t} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} \right]$

14. A fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} -5 & 3 & 0 \\ -6 & 4 & 0 \\ -6 & 6 & -2 \end{pmatrix} \mathbf{x}$ is:

Answer $\left\{ e^{t} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},\ e^{-2t} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},\ e^{-2t} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$

15. A fundamental set of solutions of $\mathbf{x}' = \begin{pmatrix} 0 & 2 & -3 \\ 2 & 0 & -3 \\ 8 & -2 & -4 \end{pmatrix} \mathbf{x}$ is:

Answer $\left\{ e^{-2t} \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix},\ e^{-t} \left[\cos 3t \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \sin 3t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right],\ e^{-t} \left[\cos 3t \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \sin 3t \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \right] \right\}$