1. Given the differential equation $x' - 2tx = 2t$. Which (if either) of the functions $x_1 = e^{t^2}$, $x_2 = e^{t^2} - 1$ is a solution?

2. Find all solutions of the first order equation $t^2 x' = x - tx$. Then find the solution that satisfies $x(1) = 1$.

3. Given the linear system $\mathbf{x}' = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix} \mathbf{x}$. Which (if any) of the vector functions

$$\mathbf{x}_1 = e^{t} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{x}_2 = e^{2t} \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \mathbf{x}_3 = e^{-2t} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

is a solution?

4. Scientists have observed that a colony of penguins on a remote Antarctic island obeys the population growth law. There were 2000 penguins in the initial population and there were 3000 penguins 2 years later.
   (i) How many penguins will there be after 8 years?
   (ii) How long will it take for the number of penguins to quadruple?

5. Four years ago a laboratory had 100 grams of a certain radioactive substance. It has 50 grams now. Suppose that a laboratory has 100 grams of the substance now.
   (i) How much will the laboratory have 10 years from now?
   (ii) How long will it take for the material to decay to 10 grams?

6. Find the equilibria of $x' = x^3 - 4x^2 + 4x$. Determine whether each equilibrium is asymptotically stable or unstable and draw a schematic of the dynamics of the equation.

7. Given the second order differential equation $x'' + a\,x' + 2x = 0$.

   (a) Determine the values of a such that the solutions are nonconstant and time periodic.

   (b) Determine the values of a such that all solutions are unbounded.

   (c) Determine the values of a such that the solutions "oscillate" and have limit 0.

8. Given the linear system $\mathbf{x}' = \begin{pmatrix} 1 & a \\ -1 & b \end{pmatrix} \mathbf{x}$. The origin $(0,0)$ is an equilibrium point.

   (a) Give values for a and b such that all solutions will circle about the origin.
   (b) Give values for a and b such that all solutions will spiral into the origin.
   (c) Give values for a and b such that the origin is a sink.
   (d) Give values for a and b such that the origin is a saddle.

9. Find the solution of the initial value problem $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$, $\mathbf{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

10. Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ 3 & -5 \end{pmatrix} \mathbf{x}$.

11. Find the general solution of the system $\mathbf{x}' = \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix} \mathbf{x}$.

12. Let $A = \begin{pmatrix} 5 & 1 \\ -6 & 0 \end{pmatrix}$, and $B = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$.

   (a) Are A and B similar?
   (b) If A is similar to B and B is similar to C, is A similar to C? Justify your answer.
   (c) Let $C = \begin{pmatrix} 2 & -1 \\ -3 & 3 \end{pmatrix}$. Are A and C similar? Justify your answer.

13. Let $B = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$. Compute e^B.