

Uniqueness of weak solutions for a fluid-structure interaction problem

GIOVANNA GUIDOBONI

(joint work with Mariarosaria Padula)

Fluid-structure interaction problems arise in many fields of science, such as aeroelasticity problems, fluttering of wings, dynamics of offshore structures subjected to the cyclic sea currents and fluid flow in compliant conduits. The main difficulty in the mathematical theory of fluid-structure interaction problems is associated to the control of the regularity of the deformable boundary whose evolution is an unknown of the problem. Few results are available in literature concerning the existence of solutions to fluid-structure interaction problems [1, 2, 3, 4, 5].

The goal of the present work is to prove uniqueness of weak solutions for a two-dimensional problem, where a layer of viscous incompressible fluid is confined between a rigid plane and a deformable structure. Periodicity is assumed in the horizontal direction and the structure is described as a linear viscoelastic beam. Existence of weak solutions for this problem was proved in [5].

The classical proof of uniqueness starts by assuming that there exists two different solutions corresponding to the same initial data. Then, energy estimates are derived for the difference between these two solutions and uniqueness follows from Gronwall's lemma. In fluid-structure interaction problems, the two solutions are defined in different domains which are unknowns of the problem as well. Therefore the classical steps of the proof need to be modified in order to give meaning to the difference between the two solutions.

Let $\mathcal{S}_1 = (\mathbf{u}_1, \eta_1)$ and $\mathcal{S}_2 = (\mathbf{u}_2, \eta_2)$ be two solutions corresponding to the same initial data, where \mathbf{u} is the velocity field of the fluid and η is the curve describing the deformable boundary. The domains in which \mathcal{S}_1 and \mathcal{S}_2 are defined are

$$\Omega_{\eta_1}(t) = \{Z = (X, Y) \in \mathbb{R}^2 \mid X \in \Sigma, 0 < Y < \eta_1(x, t)\},$$

$$\Omega_{\eta_2}(t) = \{z = (x, y) \in \mathbb{R}^2 \mid x \in \Sigma, 0 < y < \eta_2(x, t)\},$$

respectively, where Σ is the horizontal periodicity cell. A time-dependent change of coordinates is introduced to map $\Omega_{\eta_1}(t)$ onto $\Omega_{\eta_2}(t)$:

$$\begin{aligned} \phi_t : \Omega_{\eta_1}(t) &\rightarrow \Omega_{\eta_2}(t) \\ Z &\rightarrow z = \begin{cases} x = X \\ y = Y \frac{\eta_2}{\eta_1}. \end{cases} \end{aligned}$$

Therefore, every function $f(x, Y, t)$ defined on $\Omega_{\eta_1}(t)$ is transformed in the function $\hat{f}(x, Y, t) = f(x, \frac{\eta_1}{\eta_2}y, t)$ defined on $\Omega_{\eta_2}(t)$. The transformed velocity field is not solenoidal, but the vector field $\mathbf{v}_1 = J\mathbf{J}^{-1}\hat{\mathbf{u}}_1$, where \mathbf{J} is the Jacobian matrix of the transformation and J its determinant, is divergence free.

Now the difference between \mathcal{S}_1 and \mathcal{S}_2 can be defined on the same domain $\Omega_{\eta_2}(t)$ and the difference in the velocity fields will be taken as $\mathbf{u}_2 - J\mathbf{J}^{-1}\hat{\mathbf{u}}_1$ to preserve solenoidality. The regularity of the change of coordinates allows to obtain the energy estimates for the difference and uniqueness follows from Gronwall's lemma.

By introducing a slight modification in the definition of the change of coordinates, this result, as well as the existence theorem in [5], can be proved also in the case of a fluid layer contained between two deformable boundaries. This project is strongly motivated by the modeling of blood flow in large arteries, and therefore the next step is the study of existence and uniqueness of weak solutions when a time-dependent pressure drop is present.

REFERENCES

- [1] H. Beirão da Veiga, *On the existence of strong solution to a coupled fluid structure evolution problem*, J. Math. Fluid Mech. **6** (2004), 21–52.
- [2] M. Boulakia, *Existence of weak solutions for the motion of an elastic structure in an incompressible viscous fluid*, C. R. Acad. Sci. Paris, Ser. I **336** (2003), 985–990.
- [3] M. Boulakia, *Existence of weak solutions for an interaction problem between an elastic structure and a compressible viscous fluid*, C. R. Acad. Sci. Paris, Ser. I **340** (2005), 113–118.
- [4] A. Chambolle, B. Desjardins, M.J. Esteban, C. Grandmont, *Existence of weak solutions for an unsteady fluid-plate interaction problem*, J. Math. Fluid Mech. **7** (2005), 368–404.
- [5] M. Guidorzi, M. Padula, P. Plotnikov, *Galerkin method for fluids in domains with elastic walls*, submitted.